Transportation Research Part C 18 (2010) 299–310

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

A probabilistic model for traffic at actuated control signals

Francesco Viti *, Henk J. van Zuylen

Delft University of Technology, Section Traffic and Spatial Planning, Stevinweg 1, 2600 GA Delft, The Netherlands

ARTICLE INFO

Article history:
Received 27 September 2007
Received in revised form 9 February 2009
Accepted 5 May 2009

Keywords:
Vehicle actuated control
Delay
Queuing
Probabilistic modeling
Planning and design of controlled networks

ABSTRACT

Vehicle actuated controls are designed to adapt green and red times automatically, according to the actual dynamics of the arrival, departure and queuing processes. In turn, drivers experience variable delays and waiting times at these signals. However, in practice, delays and waiting times are computed at these systems with models that assume stationariety in the arrival process, and that are capable of computing simply expectation values, while no information is given on the uncertainty around this expectation. The growing interest on measures like travel time reliability, or network robustness motivates the development of models able to quantify the variability of traffic at these systems.

This paper presents a new modeling approach for estimating queues and signal phase times, based on probabilistic theory. This model overcomes the limitations of existing models in that it does not assume stationary arrival rates, but it assumes any temporal distribution as input, and allows one to compute the temporal evolution of queue length and signal sequence probabilities. By doing so, one can also quantify the uncertainty in the estimation of delays and waiting times as time-dependent processes. The results of the probabilistic approach have been compared to the results of repeated microscopic simulations, showing good agreement. The smaller number of parameters and shorter computing times required in the probabilistic approach makes the model suitable for, e.g., planning and design problems, as well as model-based travel time estimation.

1. Introduction

Traffic control at intersections is done automatically using a cyclic sequence of green, amber and red signals. Green and red times can be fixed and predetermined (*fixed time* or *pre-timed* control). An alternative is *semi-actuated* control: if a detector on the side road is activated, the main flow can be interrupted after a green period of at least the minimum green time. The green time on the minor road is determined by headway measurements, i.e., the green phase terminates when the headway between two vehicles is larger than a certain maximum time interval. Another alternative is *fully actuated control*; the mode of operation where all approaches have detectors and all green phases are controlled by means of detector information. This paper deals with this last class of signal controls.

Nowadays, traffic actuated control is most commonly applied in many countries such as, e.g., United Kingdom or the Netherlands. However, many of the existing models for estimating the performance of this control system are still based on fixed time control theory, and the effect of this adaptive mechanism is simply approximated by using discount factors (e.g., TRB, 2000). Since signal settings are not predetermined, but determined by the actual headway distribution of the arrivals, actuated control theory should differ considerably from fixed or pre-phased control in dealing with the traffic flow process.

Due to the dynamic and stochastic nature of traffic flows, different headway distributions, queue lengths and flow rates can be observed from cycle to cycle, leading to different green time extensions. If one considers that, e.g., the length of the

* Corresponding author. Tel.: +31 152786909; fax: +31 152783179.
E-mail addresses: f.viti@tudelft.nl (F. Viti), h.j.vanzuylen@tudelft.nl (H.J. van Zuylen).

0968-090X/$ - see front matter
doi:10.1016/j.trc.2009.05.003

queue depends on the number of vehicles arriving during the red phase, whose duration depends on all green time extensions given to the conflicting streams, the estimation of the expected delay experienced by a traveler becomes a complex problem already when it is limited to one signal cycle. Furthermore, an overflow queue can be observed at the end of a cycle, which will need part of the following green time(s) to be discharged. This effect determines the dynamic and stochastic behavior of these systems if looking at the cycle-to-cycle process.

This paper proposes a new computational method able to deal with the variability of traffic at actuated signals. This method provides the probability distribution of effective green and red times, and the probability distribution of the overflow queue length that can be observed at vehicle actuated controls while assuming any probability distribution of the arrivals. This information is essential for the prediction of the main characteristics of intersections controlled by actuated signals such as capacity, degree of saturation, delay, queue length, and stop rates, to quantify measures like travel time reliability and network robustness, and it can be embedded in travel time estimation models (e.g., Singh, 1998).

We focus our study on the effect of variable arrivals, therefore the probabilistic model assumes deterministic service rates and that no overflow queue is likely to be observed when green time extensions do not reach their maximum value. Together with the assigned green and red times, the probability of queue lengths at the end of each phase is computed sequentially to model the cycle-to-cycle behavior. Accounting for the time-dependent behavior due to overflow queues is of paramount importance in practice, especially when the signal operates near capacity and green times are likely to reach the maximum value. In these traffic conditions green times can be extended up to their maximum value because long overflow queues have occurred previously.

The next section presents a literature review on the existing vehicle actuated control models. A description of the probabilistic relationships, which connect arrivals, queues and signal settings, and which form the basis of the probabilistic model, is given in Section 3. The probabilistic model is later presented in Section 4 and a numerical example follows. Finally, comparison with the results of a commercial microscopic simulation program and conclusions close this paper.

2. Literature review

2.1. List of notations

t_C^*	optimal cycle length
$\bar{t}_C$	average signal cycle
$\bar{t}_g$	average effective green time for approach *i*
$\bar{t}_r$	average effective red time for approach *i*
a_i	average arrival rate for approach *i*
s_i	average service rate for approach *i*
c_i	average capacity of the approach *i*: $c_i = s_i t_{g,i}/t_C$
x_c	critical volume-to-capacity ratio
LT	lost time in the cycle
y_i	volume-to-saturation flow ratio for approach *i*
x_i	volume-to-capacity ratio for approach *i*
W	expected delay (s/pcu)
T	time interval in the Australian capacity guide delay formula
k	*k*th cycle phase
$\bar{\tau}$	unit extension, i.e., the maximum headway-time that determines the end of a green phase
$P_i[Q_i^r(k)]$	probability of vehicles queuing up during the red time period at approach *i* and during the *k*th cycle phase
$P_i[t_g^r(k)]$	probability of green time needed to serve the vehicles in queue during the red phase at approach *i* and during the *k*th cycle phase
$P_i[Q^g(k)]$	probability distribution of the maximum back of the queue for approach *i* and during the *k*th cycle phase
$P_i[t_g^g(k)]$	probability of total green time needed to fully clear the queue for approach *i* and during the *k*th cycle phase
$P_i[t_g(k)]$	probability of green time in a cycle for approach *i* and during the *k*th cycle phase
$P_i[t_g^e(k)]$	probability of assigning an extra green time extension due to short vehicle headways for approach *i* and during the *k*th cycle phase
$P_i[t_r(k)]$	probability of red time for approach *i* and during the *k*th cycle phase
$P_i[Q^o(k)]$	probability of overflow queue approach *i* at the end of the *k*th cycle phase

2.2. Classification of vehicle actuated control models

The large number of vehicle actuated control types designed in the past, and the complex architecture of such systems, partly justifies the coexistence of many approximate models and lack of a general theory. A detailed description of the actuated controller operations can be found for instance in Staunton (1976).