Computer Vision and Image Understanding 138 (2015) 61–73

Contents lists available at ScienceDirect

Computer Vision and Image Understanding

journal homepage: www.elsevier.com/locate/cviu

A statistical method for line segment detection*

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A R T I C L E I N F O

Article history:
Received 21 April 2014
Accepted 22 May 2015
Available online 30 May 2015

Keywords:
Hough transform
Statistical analysis
Endpoint specification
Fitting and interpolation

A B S T R A C T

Line segment detection is a fundamental procedure in computer vision, pattern recognition, or image analysis applications. This paper proposes a statistical method based on the Hough transform for line segment detection by considering quantization error, image noise, pixel disturbance, and peak spreading, also taking the choice of the coordinate origin into account.

A random variable is defined in each column in a peak region. Statistical means and statistical variances are calculated; the statistical non-zero cells are analyzed and computed. The normal angle is determined by minimizing the function which fits the statistical variances; the normal distance is calculated by interpolating the function which fits the statistical means. Endpoint coordinates of a detected line segment are determined by fitting a sine curve (rather than searching for the first and last non-zero voting cells, and solving equations containing coordinates of such cells).

Experimental results on simulated data and real world images validate the performance of the proposed method for line segment detection.

1. Introduction

The *Hough transform* (HT) [1–3] is widely used for extracting geometric features in images. The standard HT is inefficient with respect to run time and storage requirements; it also suffers from the peak-spreading problem in the Hough space [4,5].

In order to improve the computation and storage efficiency, many methods have been proposed, such as the *fast HT* [6], *adaptive HT* [7], special architectures *HT* [8], *probabilistic HT* [9], *randomized HT* [10], or *generalized HT* [11].

In order to locate accurately the peak, and to derive accurate normal parameters, various extensions to the standard HT have been proposed. Numerous methods put the emphasis on generating more distinct peaks by modifying the voting scheme of a Hough transform. Edge information [12–14] or image preprocessing techniques [15–17] are used to enhance the peak in the accumulator array. In addition, a continuous statistical kernel [18] is proposed for modeling the Hough votes instead of using a discrete accumulator array. This approach is computationally more expensive because it considers and models all pixels in the given image.

After finding a peak, there are three different ways for supporting the computation of accurate peak parameters:

(1) A peak cell (θ, ρ) is simply selected for specifying the normal parameters of the detected line.
(2) A weighted averaging [19,20] is used by considering neighboring cells (θ_i, ρ_i) around a peak cell.
(3) A specially designed peak localization technique is employed. For example, two accurate peak localization methods are presented in [21,22] using smoothing windows and weighted averaging, a two-stage method [23] is described by finding a median position in narrow strips passing through an initial peak, a high-accuracy HT, proposed in [24], is based on the theoretical symmetry of the *butterfly* pattern around a peak, and a subcell-accuracy method [25] locates the peak by using fitting and interpolation techniques while analyzing the butterflies.

The HT can also be used for extracting the endpoints of a detected line segment [26] although the standard HT only provides normal angle θ and normal distance ρ.

We classify HT-based methods for detecting line segments into two categories. One class of methods is based on projection [27–29]. After applying the standard HT, the detected line is cut into pairwise disjoint segments; the endpoints of one of those line segments are determined by analyzing the projection of the feature points on either the x- or y-axis in image space. Two thresholds are needed to control the length of a line segment and the width of a gap between

* This paper has been recommended for acceptance by J.-O. Eklundh.

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http://dx.doi.org/10.1016/j.cviu.2015.05.008
1077-3142/

line segments. But, how to determine these two thresholds defines then a new problem.

The second class of methods is motivated by the butterfly shape [30,31] of a peak region in Hough space [32–34]. *Butterfly features* are used to extract parameters of a line segment. The first and last cell with non-zero voting values are identified and used to compute the endpoints of the line segment by solving two sets of simultaneous equations [35–38]. However, how to obtain the first and the last cell containing a non-zero voting value is a difficult problem in the presence of image noise, pixel disturbance, and a wide line-segment.

In a recent publication [39], we described a method for detecting line segments based on minimum-entropy analysis. Direction and length of a line segment are simultaneously determined by analyzing the voting entropies. The midpoint is detected by fitting voting means. The endpoint coordinates are finally calculated based on the detected direction, length and midpoint.

This paper proposes a novel statistical method based on HT for extracting line segments. The vote in each column (around a peak) is considered to be a random variable. The normal angle and distance of a line segment are computed by fitting and interpolating statistical variances and means. Endpoint coordinates of a line segment are computed by fitting a sine curve to the statistical non-zero voting cells, computed from statistical means and variances. Thus, there is no need for searching non-zero cells and solving two equations generated from just two cells.

In distinction to [39], this paper does not apply any minimum-entropy analysis. It extracts the normal angle, the normal distance, and endpoint coordinates using statistical characteristics. This paper also proposes a solution to calculate the statistical non-zero cells instead of searching for the first and the last non-zero voting cells in each column of a butterfly.

The rest of the paper is organized as follows. Section 2 introduces a way for peak distribution analysis in Hough space. Section 3 defines the random variable and statistical characteristics. Section 4 describes our normal angle and distance detection method based on statistical characteristics. Section 5 outlines endpoint detection by fitting sine curves. Section 6 compares experimental results with simulate data and real images. Section 7 concludes.

2. Peak distribution

Our method for line segment detection focuses on the voting cells around a peak. After voting, we start with analyzing the voting value distribution in each column around a peak in the Hough space.

2.1. Hough space

A pixel (x, y) in the image space votes for a set of cells (θ, ρ) in the Hough space according to Eq. (1), defined in [2] (see comments on historic origins in [40]):

$$\rho = x \cdot \cos\theta + y \cdot \sin\theta \tag{1}$$

Votes of a set of collinear pixels in an image would create a *peak* in Hough space. When mapping a line segment from a noise-free image into the Hough space, assumed to be given at the finest quantization resolution, the peak is distinct in Hough space, and is easy to locate. However, due to various uncertainties, like image background noise, pixel disturbance, space quantization error, or line thickness, there is the phenomenon of peak spreading. See Fig. 1. The peak is not distinct anymore in Hough space. An extraction step is needed to find and locate the accurate peak parameters.

5	6	3	5	4	9	12	11	8	9	10	7	10	13	18	19	22
4	4	5	4	8	8	8	12	5	7	10	9	14	18	15	18	8
9	4	6	9	5	5	9	6	10	6	5	14	13	16	19	14	17
6	12	10	8	7	6	5	7	10	7	12	7	13	15	16	11	17
12	10	9	5	4	9	5	5	6	17	10	18	22	26	12	21	11
7	7	5	6	6	4	6	7	5	6	13	17	24	17	25	12	15
5	6	8	7	9	5	7	7	9	5	11	24	23	19	13	19	18
5	7	8	9	9	10	7	8	7	12	22	28	20	17	18	11	12
8	6	5	9	11	12	11	8	9	18	37	26	18	16	18	20	14
13	13	17	17	14	14	17	15	27	40	32	18	22	18	13	10	12
12	13	14	19	20	16	16	32	35	42	27	22	16	18	13	14	17
14	10	13	8	9	19	26	24	39	31	19	18	13	13	13	14	15
8	15	12	12	15	19	29	42	41	22	16	11	18	14	13	16	11
12	13	15	17	22	26	27	32	18	6	11	14	7	8	14	11	9
14	13	12	16	15	20	20	19	5	8	8	8	7	7	11	12	15
13	13	15	13	18	17	23	11	8	13	6	9	12	13	8	7	8
10	10	9	16	17	12	15	4	5	5	9	7	6	14	8	9	5
13	12	15	14	16	25	11	4	9	8	9	8	13	4	9	3	7
14	18	19	20	13	19	5	10	3	5	7	8	5	4	2	7	6
17	15	17	15	27	7	8	7	10	10	5	9	7	5	7	5	5
11	16	16	18	15	8	8	10	12	5	7	5	5	9	4	4	8

ρ θ

Fig. 1. Illustration of the phenomenon of peak spreading.

2.2. Peak region

Let H_{ij} be the voting value at cell (θ_i, ρ_i) in Hough space. An *initial peak* is found by using a sliding 3×3 window over all cells of the Hough space. At each position in the whole Hough space we consider the sum of all nine voting values in this placed window. If this sum is the maximum then we take the central cell of this placed window as the initial peak λ.

A larger window, centered at the initial peak, defines the *peak region*. The size of this larger window depends upon the quantization resolution of the Hough space. For example, we used a window of size 17×21 for $\Delta\theta$ equals 1°, and $\Delta\rho$ equals $1p$ (i.e. the edge length of one pixel), and a window of size 11×17 for $(\Delta\theta, \Delta\rho)$ equals $(2°, 2p)$.

2.3. Origin selection in image space

Let w and h be width and height of the image space, respectively. In the standard Hough transform, the origin in image space is selected to be at a corner of the image, identified with the origin of the coordinate system as used for representing image data. In this case, the range of θ is $[0, \pi)$, and the range of ρ in the Hough space is from $-\sqrt{w^2 + h^2}$ to $\sqrt{w^2 + h^2}$.

If the image center is selected to be the origin, the range of ρ changes to the interval

$$[-\sqrt{w^2 + h^2}/2,\ \sqrt{w^2 + h^2}/2]$$

Thus, it is reduced to half of its previous size.

The Hough transform equation is then as follows:

$$\rho = (u - w/2)\cos\theta + (v - h/2)\sin\theta \tag{2}$$

where (u, v) is the pixel location in image space.

The location of the origin in the image also influences the shape of the voting distribution of a peak region. This translation of the origin leads to a butterfly-like distribution around a peak, especially for a line segment that is far away from its perpendicular point.

Fig. 2 shows voting distributions around a peak for both placement options for the origin. There is a line segment that is far away from its perpendicular point. In case of selecting O as the origin of the image, there is only a very small number (e.g. one or two) of voting cells in many columns, and the peak is not distinct in Hough space. It is difficult to detect and to locate the peak. The likelihood of such a case is reduced by selecting the center O' as the coordinate origin, and voting values converge now "more" in the peak region. Therefore, a relatively distinct peak is generated.