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Three-dimensional volume reconstruction from slice data using phase-field models[☆]

Yibao Li[a], Jaemin Shin[b], Yongho Choi[c], Junseok Kim[c,*]

[a] School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, China
[b] Institute of Mathematical Sciences, Ewha W. University, Seoul 120-750, Republic of Korea
[c] Department of Mathematics, Korea University, Seoul 136-713, Republic of Korea

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ABSTRACT

We propose the application of a phase-field framework for three-dimensional volume reconstruction using slice data. The proposed method is based on the Allen–Cahn and Cahn–Hilliard equations, and the algorithm consists of two steps. First, we perform image segmentation on the given raw data using a modified Allen–Cahn equation. Second, we reconstruct a three-dimensional volume using a modified Cahn–Hilliard equation. In the modified Cahn–Hilliard equation, a fidelity term is introduced to keep the solution close to the slice data. The numerical methods use a hybrid method and an unconditionally stable nonlinear splitting scheme. The resulting discrete equations are solved using a multigrid method. The experiments on synthetic and real medical images are performed to demonstrate the accuracy and efficiency of the proposed method.

1. Introduction

Developing reconstruction algorithms attracts a significant amount of attention because the three-dimensional (3D) volume reconstruction from a sequence of medical images has numerous applications such as medical diagnostic, plastic and artificial limb surgery, virtual surgery system, anatomy teaching, and treatment planning [2,3]. Various algorithms have been proposed to reconstruct a surface or volume from a set of planar cross-sections. A method which combines the elastic interpolation algorithm, spline theory, and surface consistency theorem was proposed for reconstructing a smooth 3D object from serial cross sections [4]. Guo et al. presented a morphology-based mathematical method to implement the interpolation by means of a combined operation of weighted dilation and erosion [5]. Jones and Chen constructed surfaces from cross sections using a field function in each slice and the marching cubes algorithm to generate a surface consisting of polygonal facets [6]. In shape-based interpolation method, the signed distance value of a voxel to the edges of a cross section is calculated. After each slice has been assigned the distance values, distances for other slices are defined using an interpolation. Then, the volume is obtained by the zero isosurface [7]. For other approaches to 3D reconstruction, refer to [8–11].

In this paper, we present a phase-field framework for 3D volume reconstruction using slice data. The proposed algorithm has two steps: image segmentation and 3D reconstruction using two partial differential equations, which are the modified Allen–Cahn and Cahn–Hilliard equations. This paper is organized as follows. In Section 2, we describe the governing equations for the image segmentation and volume reconstruction. Section 3 describes a practically stabilized nonlinear splitting scheme for the volume reconstruction and presents a multigrid method. In Section 4, we perform numerical experiments with synthetic and real medical images. Finally, our conclusion is given in Section 5.

2. Reconstruction process

In this section, we propose a numerical algorithm for 3D volume reconstruction from slice data. We start with an illustration of the process of the proposed algorithm when we have a set of cross-sectional slice data (Fig. 1(a)).

For the first stage, by using a modified Allen–Cahn equation, the image segmentation algorithm is applied for the given slice data f to obtain the phase-field function ψ (segmented image). Fig. 1(b) shows the filled contour plots of segmented slice data ψ. For the second stage, by using a modified Cahn–Hilliard equation and the segmented slice data, we reconstruct the volume (Fig. 1(c)).

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* Corresponding author. Fax: +82 2 929 8562.
E-mail addresses: yibaoli@mail.xjtu.edu.cn (Y. Li), cfdkim@korea.ac.kr (J. Kim).
URL: http://gr.xjtu.edu.cn/web/yibaoli (Y. Li), http://math.korea.ac.kr/~cfdkim (J. Kim)

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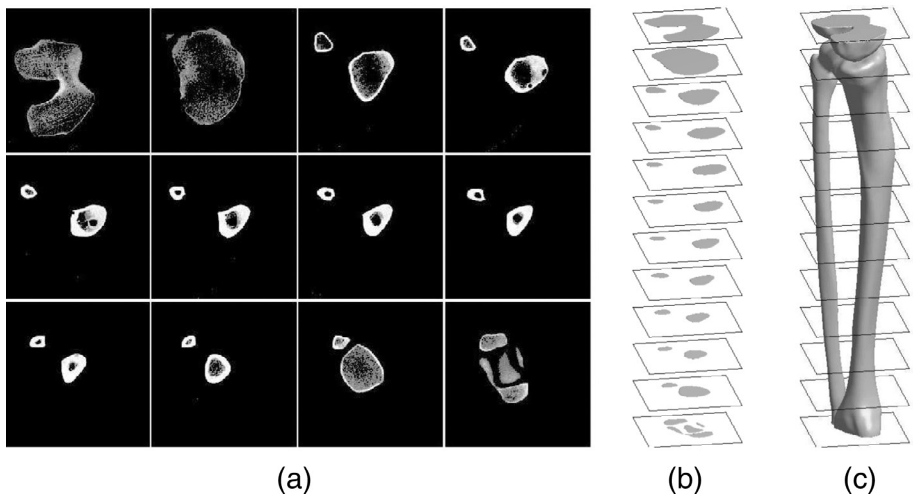

Fig. 1. Volume reconstruction from slice data: (a) given slice data, (b) filled contour plots of segmented slice data, and (c) iso-surface of the reconstructed volume.

2.1. Image segmentation: modified Allen–Cahn equation

Since the original image may have noises (Fig. 2(a) and (b)), to prepare the numerical slice data ψ (Fig. 2(c)) before applying the proposed method for volume reconstruction, we use an image segmentation algorithm [12–18]. The method we use for image segmentation is based on the Allen–Cahn equation and it enforces the diffuse interface to be the hyperbolic tangent profile. The geometric active contour model based on the mean curvature motion is given by the following evolution equation [14]:

$$\frac{\partial \psi(\mathbf{x},t)}{\partial t} = g(f_0(\mathbf{x}))\left(-\frac{F'(\psi(\mathbf{x},t))}{\epsilon^2} + \Delta\psi(\mathbf{x},t)\right) + \beta g(f_0(\mathbf{x}))F(\psi(\mathbf{x},t)), \quad (1)$$

where $\mathbf{x} = (x, y)$ and $f_0(\mathbf{x}) = (f(\mathbf{x}) - f_{\min})/(f_{\max} - f_{\min})$. Here, $f_{\max}$ and $f_{\min}$ are the maximum and minimum values of the given slice image $f(\mathbf{x})$, respectively. Here, $\psi(\mathbf{x},t)$ is a phase-field function which is close to 1 or -1. The function $g(f_0(\mathbf{x})) = 1/[1 + |\nabla(G_\sigma * f_0)(\mathbf{x})|^2]$ is the edge-stopping function, which stops the evolution when the contour reaches the edge. The function $(G_\sigma * f_0)(\mathbf{x}) = \int_\Omega G_\sigma(\mathbf{x} - \mathbf{y}) f_0(\mathbf{y}) d\mathbf{y}$ is the convolution of the given image f_0 with the Gaussian function $G_\sigma(\mathbf{x}) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$. Here, $F(\phi) = 0.25(\phi^2 - 1)^2$, ϵ is a constant that is related to the phase transition width, and β is a parameter. In this paper, we use $\sigma = 1.5$ and $\beta = 50,000$.

We apply a hybrid method [14] to solve Eq. (1) and we outline the numerical solution algorithm for the sake of completeness. We discretize Eq. (1) in $\Omega = (a, b) \times (c, d)$. Let N_x and N_y be positive even

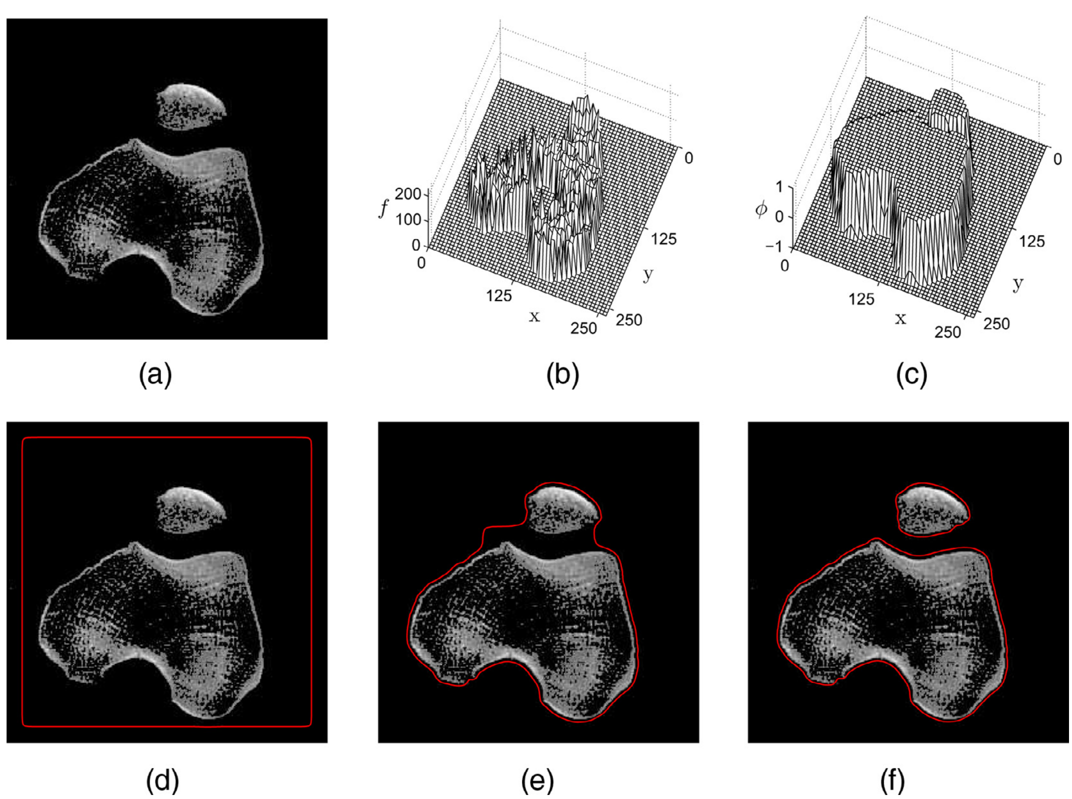

Fig. 2. Image segmentation process of the given slice data: (a) given medical image, (b) mesh plot of the given image, (c) mesh plot of the final result, and (d)–(f) evolutions of image segmentation, in which the curves are the zero contours of $\psi(x, y, t)$.