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Radial distortion invariants and lens evaluation under a single-optical-axis omnidirectional camera ☆

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ABSTRACT

This paper presents radial distortion invariants and their application to lens evaluation under a single-optical-axis omnidirectional camera. Little work on geometric invariants of distorted images has been reported previously. We establish accurate geometric invariants from 2-dimensional/3-dimensional space points and their radially distorted image points. Based on the established invariants in a single image, we construct criterion functions and then design a feature vector for evaluating the camera lens, where the infinity norm of the feature vector is computed to indicate the tangent distortion amount. The evaluation is simple and convenient thanks to the feature vector that is analytical and straightforward on image points and space points without any other computations. In addition, the evaluation is flexible since the used invariants make any a coordinate system of measuring space or image points workable. Moreover, the constructed feature vector is free of point orders and resistant to noise. The established invariants in the paper have other potential applications such as camera calibration, image rectification, structure reconstruction, image matching, and object recognition. Extensive experiments, including on structure reconstruction, demonstrate the usefulness, higher accuracy, and higher stability of the present work.

1. Introduction

Geometric invariants, reflecting intrinsic properties of objects, are extremely useful for classifying and recognizing objects [1–5]. In particular, projective geometric invariants between scene and image can be applied to recognizing objects without requiring camera calibration and complete 3-dimensional (3D) reconstruction.

In the past years, there have been many studies on projective geometric invariants under perspective cameras [1–5]. However, there are few studies on invariants under omnidirectional cameras due to the severe image distortions and the nonlinear imaging processes. The omnidirectional cameras, having a large field of view, offer great benefit to three-dimensional modeling of wide environment, robot navigation, and visual surveillance. Geometric properties of these cameras are currently being studied by a number of authors [6–18,29–37].

Catadioptric camera, fisheye camera, and wide-angle camera are all omnidirectional cameras with radial distortion. In 2005, Bayro-Corrochano and Lopez-Franco [16] projected features of the catadioptric image to the sphere defined by Geyer and Daniilidis [17], and then calculated projective geometric invariants using conformal geometric algebra, where camera intrinsic parameters should be known. Also in the same year, Wu and Hu [18] established invariant equations of space points and their radially distorted image points, in which camera optical axis position was used for 3D points and intersection point of camera optical axis with 2-dimensional (2D) scene plane was used for 2D points. Establishment of invariants without involving the optical axis knowledge in scene space or other camera parameters deserves investigations because solving these parameters is a complex task.

In this work, we:

(1) define the single-optical-axis omnidirectional camera to be a kind of omnidirectional cameras that have a single optical axis and whose optical center loci lie on the optical axis. For example, the catadioptric camera with a quadric as its mirror [17], the fisheye camera, some wide angle cameras, and the traditional perspective camera are all single-optical-axis cameras.

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(2) establish projective geometric invariant between 2D/3D space points and their radially distorted image points under a single-optical-axis camera. The invariants are called radial distortion invariants. These invariants do not involve the camera optical axis position in 3D space or the intersection point of the camera optical axis with the scene plane. Additionally, they do not involve any other camera parameters except for the principal point. The principal point can be well approximated by the center of the imaged edge contour (see the analyses in the fourth paragraph of Section 5.1 and the fourth paragraph of Section 5.2). Thus, the invariants are more practical and flexible.

(3) apply the established invariants for evaluating a single-optical-axis camera lens. We construct a criterion function and then design a feature vector. The infinity norm of this feature vector is computed which indicates the tangent distortion amount of the camera. By comparing the infinity norm with a given threshold, whether a single-optical-axis camera lens is aligned or has tangent distortion is evaluated. The algorithm is simple and convenient for evaluating a camera as the feature vector is analytical that is directly constructed from image points and space points without any other computations. In addition, the vector is free of point orders and resistant to noise. Once a camera is evaluated as no tangent distortion, only radial distortion model should be used in applications. In this paper, scene structure recovery after lens evaluation is proposed like in [18].

Geometric distortion of a camera lens includes: radial distortion, tangent distortion, or the hybrid distortion of both [29]. The distortion is an important factor for evaluating the quality of a camera lens [19–22]. However, detecting tangent distortion is difficult. Moreover, for a single-optical-axis omnidirectional camera, detection of its alignment is needed. As pointed out in [6,8,11], if the distortion center and the principal point are different for a misaligned camera, tangent distortion will appear. Thus, this paper is very useful for a single-optical-axis camera to tell whether it is aligned or has tangent distortion.

For example in Fig. 1, a catadioptric camera consisting of a quadric mirror and a perspective camera lens is a single-optical-axis omnidirectional camera. Before using this camera, alignment is needed to make the mirror face the lens rightly. In [34], Mashita, Iwai, and Yachida also think the mirror alignment is absolutely essential and think it is difficult to align the mirror and camera positions. If images of the misaligned camera were used to do camera calibration or 3D reconstruction by regarding it aligned, the results would not be accurate. How to know whether the camera is aligned or the alignment extent can be accepted? The infinity norm of the designed feature vector in this paper can be as an indication.

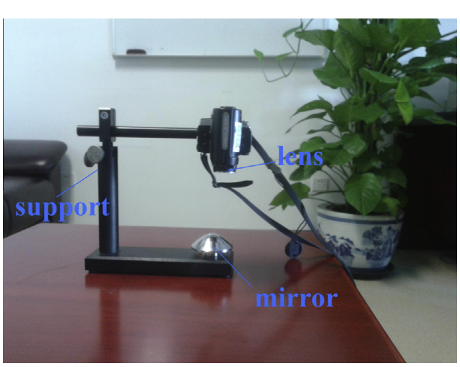

Fig. 1. A catadioptric camera consisting of a quadric mirror and a perspective camera lens: before using this camera, alignment is needed to make the mirror face the lens rightly.

Besides the proposed evaluation application, the established invariants can find other applications. For example, they can be used for recognizing polyhedrons or polygons directly from 2D distorted images without a complete 3D reconstruction like those for perspective images in [23,24].

The remainder of this paper is organized as follows. Some preliminaries are listed in Section 2. The radial distortion invariants are given in Section 3. Section 4 proposes the lens evaluation algorithm for a single-optical-axis camera. The experimental results are reported in Section 5, followed by a conclusion in Section 6.

2. Preliminaries

As we all know, a point $\boldsymbol{a}$ in a 1-dimensional (1D) space (a line) may be represented by the coordinate x, a point $\boldsymbol{B}$ in a 2D space (a plane) may be represented by the coordinates (x,y), and a point $\boldsymbol{C}$ in a 3D space may be represented by the coordinates (x,y,z). In a projective space, point representations are slightly different and they are represented by homogeneous coordinates. The homogeneous coordinates of the above point $\boldsymbol{a}$ is $s(x,1)^T$ if it is not at infinity or is $s(x,0)^T$ if it is at infinity, where s is any a nonzero scalar. Similarly, the homogeneous coordinates of $\boldsymbol{B}$ is $s(x,y,1)^T$ or $s(x,y,0)^T$ and of $\boldsymbol{C}$ is $s(x,y,z,1)^T$ or $s(x,y,z,0)^T$. In the following of this paper, a bold italic letter just denotes a point or its homogeneous coordinates and sometimes a vector or a matrix.

We use the symbol "| |" to denote determinant of points in it. For example, $|\boldsymbol{a}_1\boldsymbol{a}_2|$ is the determinant of 1D finity points $\boldsymbol{a}_i$, $i = 1, 2$ with homogeneous coordinates s_i $(x_i,1)^T$, whose absolute value is also the distance between $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ if both s_i are taken as 1. $|\boldsymbol{B}_1\boldsymbol{B}_2\boldsymbol{B}_3|$ is the determinant of 2D finity points $\boldsymbol{B}_i$, $i = 1,2,3$ with homogeneous coordinates $s_i(x_i,y_i,1)^T$. $|\boldsymbol{C}_1\boldsymbol{C}_2\boldsymbol{C}_3\boldsymbol{C}_4|$ is the determinant of 3D finity points $\boldsymbol{C}_i$, $i = 1, 2, 3, 4$ with homogeneous coordinates $s_i(x_i,y_i,z_i,1)^T$. For notational convenience, if there is no risk of ambiguity, $|\boldsymbol{B}_1\boldsymbol{B}_2\boldsymbol{B}_3|$ for 2D points $\boldsymbol{B}_i$ will be simply written as $|\boldsymbol{B}_{1,2,3}|$ and $|\boldsymbol{C}_1\boldsymbol{C}_2\boldsymbol{C}_3\boldsymbol{C}_4|$ for 3D points $\boldsymbol{C}_i$ as $|\boldsymbol{C}_{1,2,3,4}|$.

The cross ratio is fundamental in projective geometry that keeps invariant under a projective transformation [25]. For four collinear points $\boldsymbol{a}_i$, $i = 1 \dots 4$ being 1D homogeneous coordinates, the cross ratio is defined as

$$|\boldsymbol{a}_1\boldsymbol{a}_3||\boldsymbol{a}_2\boldsymbol{a}_4|/(|\boldsymbol{a}_2\boldsymbol{a}_3||\boldsymbol{a}_1\boldsymbol{a}_4|). \tag{1}$$

In a 2D projective plane, a pencil of lines is a set of lines, each of which passes through a fixed point. The fixed point is called the vertex of the pencil. There is a cross ratio from a pencil of four lines, which is equal to the cross ratio of four collinear intersection points of a general transversal line with this pencil. As shown in Fig. 2, the four lines $\boldsymbol{A}_0\boldsymbol{A}_i$, $i = 1 \dots 4$ construct a pencil with $\boldsymbol{A}_0$ being the vertex. This pencil is denoted as $\boldsymbol{A}_0(\boldsymbol{A}_1,\boldsymbol{A}_2,\boldsymbol{A}_3,\boldsymbol{A}_4)$ and its cross

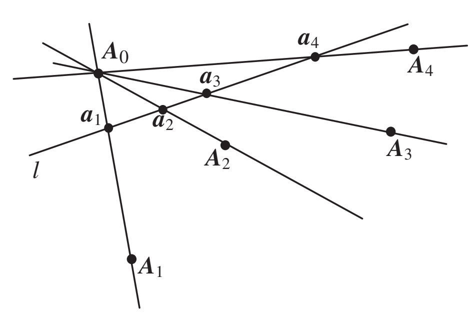

Fig. 2. A pencil of lines.