Computer Vision and Image Understanding 126 (2014) 28–37

Contents lists available at ScienceDirect

Computer Vision and Image Understanding

journal homepage: www.elsevier.com/locate/cviu

The reconstructed residual error: A novel segmentation evaluation measure for reconstructed images in tomography

Tom Roelandts [a,*], K. Joost Batenburg [a,b,c], Arnold J. den Dekker [a,d], Jan Sijbers [a]

[a] iMinds-Vision Lab, University of Antwerp, Universiteitsplein 1, 2610 Wilrijk, Belgium
[b] Centrum Wiskunde & Informatica, Science Park 123, 1098 XG Amsterdam, The Netherlands
[c] Mathematical Institute, Leiden University, P.O. Box 9512, 2300 RA Leiden, The Netherlands
[d] Delft Center for Systems and Control, Delft University of Technology, 2628 CD Delft, The Netherlands

ARTICLE INFO

Article history:
Received 18 February 2014
Accepted 21 May 2014
Available online 4 June 2014

Keywords:
Computed tomography
Image segmentation
Segmentation evaluation

ABSTRACT

In this paper, we present the reconstructed residual error, which evaluates the quality of a given segmentation of a reconstructed image in tomography. This novel evaluation method, which is independent of the methods that were used to reconstruct and segment the image, is applicable to segmentations that are based on the density of the scanned object. It provides a spatial map of the errors in the segmented image, based on the projection data. The reconstructed residual error is a reconstruction of the difference between the recorded data and the forward projection of that segmented image. The properties and applications of the algorithm are verified experimentally through simulations and experimental micro-CT data. The experiments show that the reconstructed residual error is close to the true error, that it can improve gray level estimates, and that it can help discriminating between different segmentations.

1. Introduction

In many applications of tomography [1], such as the delineation of anatomical structures (in medical imaging) and object detection (in computer vision), the reconstructed image must be segmented before the results can be analyzed. Segmentation is defined as the classification of image pixels into distinct classes, based on similarity with respect to some characteristic. Numerous methods have been proposed, such as global and local thresholding, region growing, clustering, and atlas-guided approaches [2–4].

Given a segmentation, objectively evaluating the accuracy of that segmentation is not a trivial task [5,6]. *Supervised* methods evaluate a segmentation algorithm by comparing its output with *gold standard* segmentations. However, since such segmentations are typically not available in practice, they must often be generated manually, which is not easy and may make the evaluation subjective. *Unsupervised* methods (also sometimes called *stand-alone methods* [7]) do not need gold standards, as they evaluate the segmentation results directly, using one or more of the *de facto* standard criteria of Haralick and Shapiro [2]. These methods are objective and applicable to a wide variety of images, but their analysis is restricted to the segmentation result itself. A possibility that is often overlooked, in the specific case of tomography, is exploiting the available projection images, which can provide external information about the segmented image.

The current paper introduces the *reconstructed residual error*, which *does* exploit the original projection images. Our method is applicable to reconstruction problems for which the segmentation is based on the density of the scanned object, where we use the term *density* to refer to the particular physical property of the object of which linear projections are acquired during the scanning process (e.g., mass density, X-ray attenuation, electron beam scattering, etc.). The reconstructed residual error is an unsupervised evaluation in the terminology of [6], since it is an objective evaluation at the level of the segmentation itself that does not need a reference image [6, Fig. 1]. In contrast to the unsupervised methods surveyed in [6], the proposed method does not have to rely on the criteria of Haralick and Shapiro, since it uses the projection images as external information.

The reconstructed residual error evaluates a given segmentation by providing a spatial map of the errors. It is computed by reconstructing the difference between the recorded data and the forward projection of that segmentation. The computation of the error map is independent of the methods that were used for reconstructing the image and determining the segmentation.

The remainder of this paper is organized as follows. In Section 2, the reconstructed residual error is defined and its properties are described in detail. Section 3 reports on the results of experiments, using both simulations and experimental micro-CT data. These

* Corresponding author.
E-mail address: tom.roelandts@uantwerpen.be (T. Roelandts).

http://dx.doi.org/10.1016/j.cviu.2014.05.007
1077-3142/

results are discussed in Section 4, and conclusions are drawn in Section 5.

2. Reconstructed residual error

Here, we describe the reconstructed residual error. We first present an intuitive overview of its computation, before giving a complete description of its properties.

2.1. Overview

Fig. 1 presents an overview of the computation of the reconstructed residual error. The example is based on a two-dimensional slice through objects with simple geometrical shapes and only two different gray levels (Fig. 1a). The original projection data (Fig. 1b) is a sinogram, acquired by rotating around the object. This sinogram is then reconstructed (Fig. 1c) and segmented (Fig. 1e). For this overview figure, the segmented reconstruction was computed by thresholding Fig. 1c, and subsequently choosing gray levels. The reconstructed residual error is computed from the original projection data (Fig. 1b) and the segmented reconstruction (Fig. 1e). The original (non-segmented) reconstruction (Fig. 1c) is not used.

To compute the reconstructed residual error, the segmented reconstruction (Fig. 1e) is first projected forward. The result of this operation (Fig. 1d) is then subtracted from the original projections, resulting in the *residual projection error* (Fig. 1g). The residual projection error is then reconstructed to provide the reconstructed residual error (Fig. 1h), which provides a spatial map of the segmentation error. From Fig. 1h, it is clear that both gray levels of the segmented reconstruction are incorrect. The erroneous lines and dots in the segmented reconstruction (Fig. 1e), which are caused by artifacts in the reconstruction (Fig. 1c), are also clearly visible in Fig. 1h. Note that the true error (Fig. 1i), which is the difference between the original object (Fig. 1a) and the segmented reconstruction, is quite close to the reconstructed residual error.

An alternative that might be considered, is to simply compute the difference between the original and the segmented reconstruction (Fig. 1f). However, this difference can be expected to show much more reconstruction artifacts, as is explained in Section 2.4. An intuitive way to see this is that, for phantom experiments, the segmented reconstruction can potentially be identical to the phantom, in which case the residual projection error (Fig. 1g) would be zero. Hence, a perfect segmentation would result in the reconstructed residual error (Fig. 1h) being zero everywhere, while the difference between the original and the segmented reconstruction (Fig. 1f) will always exhibit the reconstruction artifacts of the original reconstruction (Fig. 1c).

2.2. Notation and concepts

The projection process in tomography can be modeled as a linear operator that is determined by the projection geometry. This leads to a system of linear equations,

$$\boldsymbol{Wx} + \boldsymbol{n} = \tilde{\boldsymbol{p}}, \tag{1}$$

where $\tilde{\boldsymbol{p}} \in \mathbb{R}^m$ is the measured projection data, $\boldsymbol{n} \in \mathbb{R}^m$ is the noise, and $\boldsymbol{x} \in \mathbb{R}^n$ is the unknown image. The linear operator is represented by the $m \times n$ matrix $\boldsymbol{W}$, the *projection matrix*. An approximate solution $\hat{\boldsymbol{x}} \in \mathbb{R}^n$ of (1) can then be computed, for example by minimizing some norm $\|\boldsymbol{Wx} - \tilde{\boldsymbol{p}}\|$.

In Sections 2.4 and 2.5, we assume that $\boldsymbol{n} = \boldsymbol{0}$ and explicitly refer to the ground truth object as $\boldsymbol{g} \in \mathbb{R}^n$, which leads to $\boldsymbol{p} = \boldsymbol{Wg}$, where $\boldsymbol{p} \in \mathbb{R}^m$ are noiseless projections. We denote a general reconstruction algorithm as an operator $\mathcal{R} : \mathbb{R}^m \to \mathbb{R}^n$, which leads to $\hat{\boldsymbol{x}} = \mathcal{R}\boldsymbol{Wg}$. The *so-called* shortest least squares solution is denoted as $\boldsymbol{x}^+$, where $\boldsymbol{x}^+ = \boldsymbol{W}^+\boldsymbol{p}$, with $\boldsymbol{W}^+$ the Moore–Penrose pseudoinverse [8, Section 7.3] of $\boldsymbol{W}$. We denote the *row* (or *image*) *space* of $\boldsymbol{W}$ by $R(\boldsymbol{W})$, and the *null space* by $N(\boldsymbol{W})$. See again [8] for details on these concepts from linear algebra.

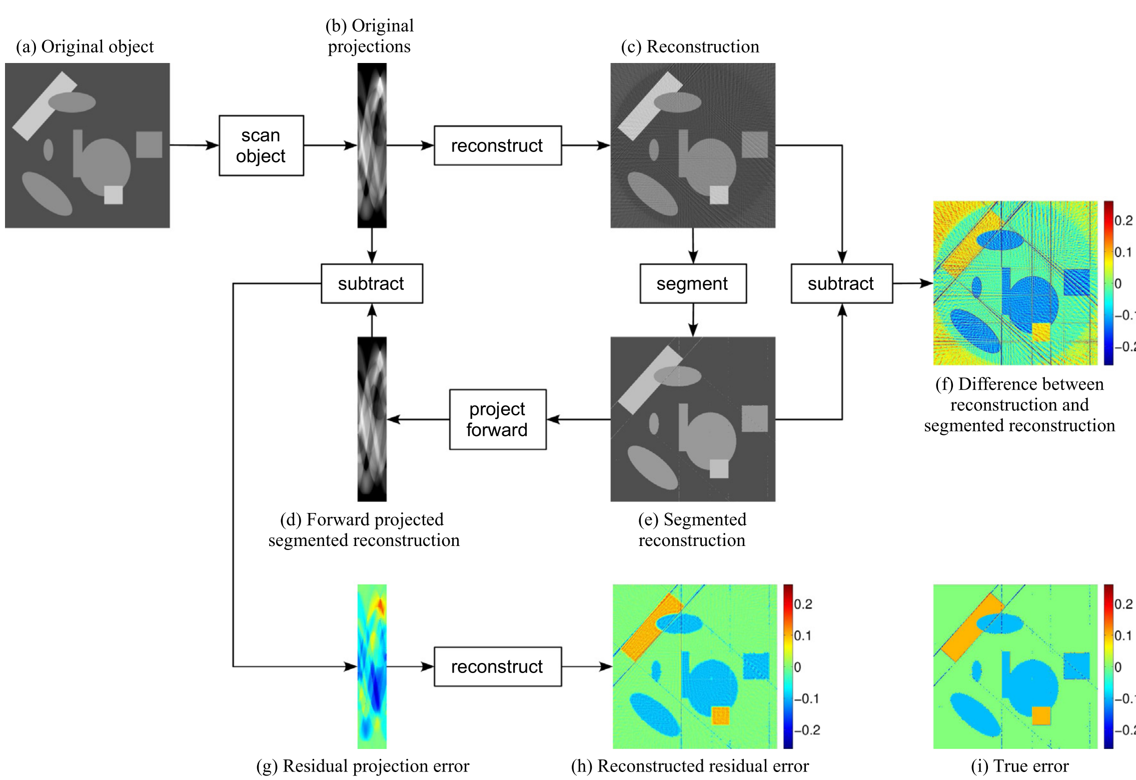

Fig. 1. Overview of the computation of the reconstructed residual error.