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Unitary torus model for conical mirror based catadioptric system [☆]

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ABSTRACT

Catadioptric systems consist of the combination of lenses and mirrors. From them, central panoramic systems stand out because they provide a unique effective viewpoint, leading to the well-known unifying theory for central catadioptric systems. This paper considers catadioptric systems consisting of a conical mirror and a perspective camera. Although a system with conical mirror does not possess a single projection point, it has some advantages as the cone is a very simple shape to produce, it has higher resolution in the peripheral, and adds less optical distortion to the images. The contributions of this work are the model of this non-central system by means of projective mappings from a torus to a plane, the procedure to calibrate this system, and the definition of the conical fundamental matrix with a role similar to that of perspective cameras. Additionally, a procedure to compute the relative motion between two views from the conical fundamental matrix is presented. The proposal is illustrated with simulations and real experiments.

1. Introduction

Vision systems stand out from other types of sensors because they provide very rich information and because of their versatility and low cost. For the last years, the use of omnidirectional cameras has been growing because they provide a panoramic view from a single image. A main class of cameras are the catadioptric systems, consisting of the combination of lenses and mirrors. Single viewpoint is a desirable property of a camera system, and the complete class of central catadioptric sensors with one mirror and lens are treated in [1]. A unifying theory for all central catadioptric systems was proposed in [2] and extended in [3]. In these works, the image formation model is developed by defining the well-known unified sphere model. Usual central catadioptric cameras are built combining a hyperbolic mirror with a perspective camera placed on one of the foci, or a parabolic mirror with an orthographic camera. Although having a single viewpoint is a desirable requirement in design, other features may be considered depending on the application.

This paper considers catadioptric systems using a conical mirror and a perspective camera. Three example images taken with a conventional camera, a camera system with hyperbolic mirror, and the conical mirror can be compared in Fig. 1. These images have been taken in the same environment, a square outdoors, to illustrate their different features. The best quality but narrowest field of view is given by the conventional camera. The hyper-catadioptric camera captures the camera itself and shows good quality around the camera system, while the rest of the environment is concentrated in the border of the image with low resolution. In this case, part of the border is filled with the sky. On the other hand, the conical mirror based camera system does not capture the sky or the bottom part of the camera system and shows good resolution for the rest of the scene (i.e. the part of the scene between the sky and the floor around the camera system). For the same hardware, different setups lead to quite different image results (For instance: camera zoom, camera-mirror distance…).

In general, the advantages and disadvantages of each different system have to be evaluated depending on the application considered. In particular, some of the advantages of conical mirror based cameras compared to usual catadioptric systems are that the cone is a very simple shape to produce, it has higher resolution in the peripheral and adds less optical distortion to the images [4]. Another advantage compared to paracatadioptric systems is that a perspective camera is used instead of an expensive and complex orthographic camera. The use of conical mirror results in a non-central camera system, which is easier to produce with respect to the central catadioptric model because the latter requires precise alignment of the optical center. Actually, when the viewpoint of the perspective camera coincides with the vertex of the conical mirror, a central camera system is obtained [1]. This particular case has been studied in [4,5] showing its feasibility but reducing the

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Fig. 1. Examples of images taken with a standard camera (left), hyper-catadioptric camera (center) and with conical mirror camera system (right).

configuration possibilities. Thus, despite non-centrality, the versatility of the conical mirror setup is a desirable property as shown for example in the single mirror stereo arrangement proposed in [6]. The field of view of the conical mirror system is smaller in general but more flexible. In fact, the smaller vertical field of view can be seen as an advantage because it provides higher angular resolution with the same number of pixels. For example, the camera is out of its field of view and it is not projected in the image plane (so part of the image is not wasted imaging the sensor itself) [7]. Application examples using a conical mirror based system was presented in [8], providing a method for mobile robot navigation that avoids collisions with objects, and in [9], where conical mirrors are studied as radial imaging systems to recover the 3D structure of an object.

Alternatively to catadioptric systems, panoramic images may also be captured by sensor-line cameras through rotation [10]. In the case in which the optical center follows a circular path with the line sensor parallel to the rotating axis, a panoramic image can be captured on a cylindrical surface. This case resembles the image formation model of the conical mirror based camera because its viewpoint is also a circular locus. The main benefit of these sensor-line cameras is the high resolution of recorded image data, whereas some disadvantages are the mechanical complexity of the rotating system and, since the acquisition of lines may require some time, moving objects in the recorded scene will appear geometrically distorted.

As said, the conical mirror and camera system is non-central, and the contribution of this work is the projection model for this system. This is achieved by extending the concept of unitary sphere model to the unitary torus model and taking into account that the viewpoint of this system lies on a circular locus. Different methods have been proposed in the literature for the calibration of central catadioptric systems [11], for example by using line images [12] or point images from planar grids [13,14], and also for hybrid central cameras [15]. Methods to calibrate generalized cameras have been formulated capturing both central and non-central cameras in a unified framework [16,17]. Additionally, non-central systems with revolution symmetry can be studied as axial cameras since all the light rays pass through the axis of the symmetry [18]. Regarding non-central systems based on conical mirror, a method to calibrate the omnidirectional conical-based sensor named SYCLOP was presented in [19]. In that work, the different transformations between the world object, the conical mirror, and image plane were developed to calibrate the system by using a two-plane calibration pattern. Here, we propose a procedure for calibrating the conical mirror system by using the proposed unitary torus model.

The estimation problem of the epipolar geometry using omnidirectional vision has been studied for central cameras [20,21] or approximately central cameras [22]. In [23], the existence of a general 15×15 fundamental matrix for all central catadioptric cameras is shown. Multi-view geometry is investigated in [24] considering a highly general imaging model using Plücker coordinates for central or non-central camera types. The epipolar geometry has been investigated for linear pushbroom cameras [25], for crossed-slits projection [26], and for circular panoramas [27]. In this work the epipolar geometry of a non-central catadioptric system based on a conical mirror is considered, and the conical fundamental matrix is defined with a role similar to the fundamental matrix of perspective cameras. The procedure to estimate the conical fundamental matrix from point correspondences is presented. As application, camera motion across two views can be obtained from this fundamental matrix.

The contributions of this work are the model of the non-central system by means of projective mappings from a torus to a plane, the procedure to calibrate this system, and the definition of the conical fundamental matrix ($\mathbf{F_c}$) with a role similar to that of perspective cameras. Additionally, a procedure to compute the relative motion between two views from the conical fundamental matrix is presented. The model has the advantage of computational simplicity to deal with the imaging theory of the conical mirror. The computation of model parameters and the determination of the relative camera placement of two or more cameras is also easier. This paper extends the work presented in [28] with more details on the model and the conical fundamental matrix, adding also a procedure for the catadioptric camera calibration. The previous paper was illustrated with simulations whereas the present work is also tested with the real catadioptric system.

These contributions are novel regarding the state of the art given that most of the literature on the topic of conical mirror based camera systems focuses on modeling particular cases (e.g. like the projection of radial straight lines), and only a few works study the whole general system. Contrary to the procedure used so far with cameras based on conical mirror, where the camera system is calibrated by using the laws of reflection for describing the system projection (e.g. [19]), the unitary torus allows calibrating a simple model encapsulating the projection geometry. Additionally, further analysis into the multiple view geometry of this camera system involved with general motions has not been fully considered yet. With the proposed torus model, the camera system can be modeled with projections from the torus to the image plane. An advantage is that thanks to this unitary torus model, the multiple view geometry is formulated directly with point correspondences on the torus, rather than with correspondences between associated camera rays in 3D, as proposed for example in [24] for general cameras. This proposed formulation of the physical model leads us to the definition of the epipolar curves and the conical fundamental matrix, which can be easily computed from point correspondences on the torus. In this sense, the model we propose brings similar advantages to the non-central conical mirror based camera system than the sphere model to central systems. In this context, another advantage of the torus model is that the motion parameters can be easily extracted from the conical fundamental matrix.

The paper is organized as follows. Section 2 proposes the camera model with a conical mirror and the procedure to calibrate this catadioptric system. The conical fundamental matrix is derived in Section 3. Section 4 presents the method to compute the relative