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A discrete dynamical system of formulating traffic assignment: Revisiting Smith's model

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ABSTRACT

Through relaxing the behavior assumption adopted in Smith's model (Smith, 1984), we propose a discrete dynamical system to formulate the day-to-day evolution process of traffic flows from a non-equilibrium state to an equilibrium state. Depending on certain preconditions, the equilibrium state can be equivalent to a Wardrop user equilibrium (UE), Logit-based stochastic user equilibrium (SUE), or boundedly rational user equilibrium (BRUE). These equivalence properties indicate that, to make day-to-day flows evolve to equilibrium flows, it is not necessary for travelers to choose their routes based on actual travel costs of the previous day. Day-to-day flows can still evolve to equilibrium flows provided that travelers choose their routes based on estimated travel costs which satisfy these preconditions. We also show that, under a more general assumption than the monotonicity of route cost function, the trajectory of the dynamical system converges to a set of equilibrium flows by reasonably setting these parameters in the dynamical system. Finally, numerical examples are presented to demonstrate the application and properties of the dynamical system. The study is helpful for understanding various processes of forming traffic jam and designing an algorithm for calculating equilibrium flows.

1. Introduction

Traffic assignment, also known as route assignment or route choice, is the fourth step of conventional transportation planning, following trip generation, trip distribution, and mode choice. Its aim is to find link or path flow patterns in a traffic network, given the network topology, origin-destination (OD) trip rates, and link performance functions. Its result is an estimate of traffic volume on each link in the network and the associated measures of system performance. The measures may be the system's total travel time or cost, traffic safety, fuel consumption, environmental pollution, etc.

Executing traffic assignment requires the specification of a rule by which travelers choose their traveling routes. It is reasonable to assume that every traveler will try to minimize his or her own travel cost when traveling from an origin to a destination. A stable state termed as the user equilibrium (UE) state is reached when all routes actually used for a specific OD pair have equal travel cost which is less than or equal to those on any of the unused routes. Since individual travelers are expected to behave independently, at the UE state no traveler has incentive to change his or her traveling route. The concept of the UE was first proposed by Wardrop (1952).

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Subsequently, the traffic assignment is extended to stochastic traffic assignment and the UE criterion is extended to stochastic user equilibrium (SUE) criterion. With the assumption that the route choice of travelers is based on perceived rather than measured travel costs and the travel costs perceived by travelers are random variables, a SUE model was first formulated by Daganzo and Sheffi (1977). At the SUE, no traveler can improve his or her perceived travel cost by unilaterally changing routes. For a comprehensive review about UE and SUE, readers may refer to Sheffi (1985).

In the late 1980s, Mahmassani and Chang (1987) introduced the concept of bounded rationality, which early is a hot topic in economic literatures, into transportation field and proposed the boundedly rational user equilibrium (BRUE) in the sense that travelers may choose a route with non-minimum cost if the travel cost saving by switching to a route with minimum cost is not large enough. At the BRUE state, travelers can select any route whose travel cost is within an indifference band of the minimum route cost between each OD pair, i.e., the travel cost of each route used by travelers is not more than the minimum travel cost by a certain value. Szeto and Lo (2006) introduced the BRUE into dynamic traffic assignment, gave a concept of so-called boundedly rational dynamic user equilibrium (BR-DUE), and proposed a mathematical model formulated as a nonlinear complementarity problem whose solutions correspond to BR-DUE. Lou et al. (2010) first formulated and analyzed static BRUE problem for general networks. Han et al. (2015) provided a complete theory of BR-DUE, including the formulation of mathematical models, properties of the solution set, and computation of solutions with convergent algorithms. Zhao and Huang (2016) first verified the boundedly rational route choice behavior through conducting a laboratory experiment and then proposed a BRUE model under satisficing rule.

It is well known that, in realistic traffic networks, travelers adjust their traveling routes from day to day with their experiences or information provided by an advanced traffic information system (ATIS), and the resultant link or route flows evolve over days before reaching an equilibrium state. Even if a traffic system already reached an equilibrium state, the traffic flow pattern would probably fall into a disequilibrium state due to the perturbation of controlled inputs, exogenous information or random events, and then start to adjust again towards a new equilibrium. This leads to growing interests in studying the evolution process of traffic flows towards an equilibrium state or the day-to-day dynamics of route choices in the last three decades. For example, Cantarella (2013) and Bifulco et al. (2014, in press) modeled the day-to-day dynamics of route choices in a traffic network equipped with intelligent transportation system (ITS) and evaluated the performance of ITS in the context of day-to-day dynamics. It is worth mentioning that the day-to-day dynamics refers to system variations occurring between successive reference periods, which can be either the whole day or part of the whole day, e.g., the morning peak period (Cascetta and Cantarella, 1991; Cantarella and Cascetta, 1995). Exploration of the day-to-day dynamics is useful for better understanding various processes of forming traffic jam and better using various ATIS from either a theoretical or practical standpoint.

Moreover, exploration of the day-to-day dynamics opens up another avenue for improving traveling utility (e.g., decreasing traveling cost or increasing link capacity) in traffic networks. Both road pricing and signal control have been recognized as two classes of effective instruments of improving traveling utility (e.g., Yang and Huang, 2005; Boyles et al., 2010; de Palma and Lindsey, 2011; Lawphongpanich and Yin, 2012; Zheng et al., 2016; Liu et al., 2015; Yang and Jayakrishnan, 2015; Yang et al., 2015; Kutadinata et al., 2016). However, most previous studies of the two classes of instruments are based on stationary equilibrium flow patterns in traffic networks. Recently, researchers developed dynamic road pricing and signal control schemes in the context of the day-to-day adjustment process of traffic flows (e.g., Friesz et al., 2004; Yang et al., 2004, 2010; Yang et al., 2007; Tan et al., 2015; Bifulco et al., 2014; Liu and Smith, 2015; Smith, 2015; Smith et al., 2015; Xiao and Lo, 2015). In these dynamic schemes, either the cumulative travel utility over a planning period, taking into account the day-to-day dynamics, is optimized through controlling the road pricing (or signal time) on each day, or the road pricing (or signal time) on each day is determined by known or revealed information on previous days so that traffic flows are forced to evolve to a state with higher traveling utility.

The day-to-day dynamics of route choices towards the Wardrop UE state can be formulated as either a dynamical system with route-based variables or one with link-based variables, which governs the dynamic evolution of route flows or link flows towards the Wardrop UE state, respectively. Yang and Zhang (2009) classified the route-based dynamical systems into five major categories, i.e., the simplex gravity flow dynamics (e.g., Smith, 1983), the proportional-switch adjustment process (e.g., Smith, 1984; Smith and Wisten, 1995; Huang and Lam, 2002; Peeta and Yang, 2003), the network tatonnement process (e.g., Friesz et al., 1994, 2004; Jin, 2007), the projected dynamical system (e.g., Zhang and Nagurney, 1996; Nagurney and Zhang, 1997), and the evolutionary traffic dynamics (e.g., Sandholm, 2001; Wang et al., 2013; Tan et al., 2015). He et al. (2010) formulated a day-to-day traffic assignment model that directly deals with link flow variables. Other link-based dynamical system models may refer to Smith and Mounce (2011), He and Liu (2012), etc. All those route-based and link-based models, mentioned above, satisfy the requirement of rational behavior adjustment process (RBAP) (Zhang et al., 2001).

Guo et al. (2013) and Guo et al. (2015) proposed the discrete and continuous rational adjustment processes of link flows, respectively, and analyzed some mathematical properties of these processes. Of course, there exist other day-to-day traffic assignment models whose stationary states correspond to the SUE state (e.g., Cantarella and Cascetta, 1995; Cantarella, 1997, 2013; Watling, 1999; Watling and Hazelton, 2003; Bie and Lo, 2010; Smith and Watling, 2016), the BRUE state (e.g., Hu and Mahmassani, 1997; Guo and Liu, 2011; Guo, 2013; Wu et al., 2013), or an equilibrium state with tradable mobility credits (Ye and Yang, 2013). Readers may refer to Watling and Cantarella (2013, 2015) for both synthesis and development of the day-to-day dynamics of route choice.

In this paper, we revisit the proportional-switch adjustment process, which was first proposed by Smith (1984). Up to now, the process has been extended mainly in four aspects. First, some mathematical properties of the process, for example,