Computer Vision and Image Understanding 114 (2010) 322–333

Contents lists available at ScienceDirect

Computer Vision and Image Understanding

journal homepage: www.elsevier.com/locate/cviu

A quadratic programming based cluster correspondence projection algorithm for fast point matching

Wei Lian [a,b], Lei Zhang [a,*], Yan Liang [b], Quan Pan [b]

[a] Biometric Research Center, Dept. of Computing, The Hong Kong Polytechnic University, Hong Kong
[b] College of Automation, Northwestern Polytechnic University, Xi'an, China

ARTICLE INFO

Article history:
Received 2 December 2007
Accepted 3 December 2009
Available online 11 December 2009

Keywords:
Point matching
Quadratic programming
POCS
Clustering

ABSTRACT

Point matching is a challenging problem in the fields of computer vision, pattern recognition and medical image analysis, and correspondence estimation is the key step in point matching. This paper presents a quadratic programming based cluster correspondence projection (QPCCP) algorithm, where the optimal correspondences are searched via gradient descent and the constraints on the correspondence are satisfied by projection onto appropriate convex set. In the iterative projection process of the proposed algorithm, the quadratic programming technique, instead of the traditional POCS based scheme, is employed to improve the accuracy. To further reduce the computational cost, a point clustering technique is introduced and the projection is conducted on the point clusters instead of the original points. Compared with the well-known robust point matching (RPM) algorithm, no explicit annealing process is required in the proposed QPCCP scheme. Comprehensive experiments are performed to verify the effectiveness and efficiency of the QPCCP algorithm in comparison with existing representative and state-of-the-art schemes. The results show that it can achieve good matching accuracy while reducing greatly the computational complexity.

1. Introduction

Image registration and shape matching are fundamental yet challenging problems in different application domains including computer vision, pattern recognition and medical imaging [1–3]. By using the extracted feature points to represent the original image or shape, the registration problem can be formulated as a point matching problem. There are usually two unknown variables involved: the transformation and the correspondence [8]. Compared with the transformation, the correspondence tends to be more difficult to handle because it is constrained and there are a large number of elements in it.

To tackle the problem, different approaches have been proposed. Among them, the robust point matching (RPM) algorithm and its variants [8–10,12] are very successful. The softassign technique [12] allows fuzzy or partial matches between two point sets. The deterministic annealing technique [13] is used to control the fuzziness and optimize gradually the correspondence. The Sinkhorn technique is used to enforce the constraints on the correspondence by alternate row and column normalization [14]. The result achieved by RPM is generally satisfactory but the annealing process used in it is explicit and needs to be manually controlled.

Graph matching is a problem sharing similarities with point matching. It focuses on recovering the correspondence between two sets of nodes or edges in two graphs. Similar to RPM, the graduated assignment (GA) algorithm [15] relaxes the correspondence to be continuously valued and uses deterministic annealing for optimization. The successive projection graph matching (SPGM) algorithm [20] improves GA by replacing the annealing scheme with gradient descent based method for correspondence recovery. The constraint for correspondence is satisfied by constrained projection.

Inspired by the optimization scheme of SPGM, we propose a new point matching method in this paper. Point matching is formulated as a joint linear assignment-least square optimization problem in the same way as in [8]. The value of the energy function is decreased by solving a least square problem w.r.t. the transformation and gradient descent w.r.t. the correspondence. The constraint for correspondence is satisfied by constrained projection. Compared with RPM, no annealing schedule is required to explicitly control the fuzziness of the correspondence.

The constrained projection in [20] and our problem setting have no closed-form solution. The algorithm proposed in [20] is based on POCS technique, which implements by decomposing the original constraint set into two simpler subsets, for which the closed-form solutions exist. The solution to the original projection problem is approximated by successive projections onto the two subsets.

* Corresponding author.
E-mail address: cslzhang@comp.polyu.edu.hk (L. Zhang).

1077-3142/$ - see front matter © 2009 Elsevier Inc. All rights reserved.
doi:10.1016/j.cviu.2009.12.001

The above method has some inherent problems. First, the correspondence to be projected is only used for initialization but it is not exploited in the rest of the procedure. Therefore, accumulation error may occur in the computed solution. Second, certain portion of information contained in the correspondence to be projected may become lost due to the decomposition of the original constraint set into simpler subsets. These problems degrade the overall performance of the method.

To overcome these problems, we propose a quadratic programming (QP) based method for constrained projection based on the fact that constrained projection is equivalent to a quadratic program. However, QP may become computationally inefficient when the cardinalities of two point sets become large. To remedy this problem, we further introduce the idea of clustering into the projection process to reduce the number of variables to be solved in QP. The resulting quadratic programming based cluster correspondence projection (QPCCP) algorithm is empirically proved to converge faster, have higher computationally efficiency and higher registration accuracy than POCS based method.

The rest of the paper is organized as follows. Section 2 briefly reviews the related works. Section 3 formulates the point matching problem considered in this paper. Section 4 presents the framework of gradient based point matching. Section 5 describes the QPCCP algorithm. Section 6 focuses on the choice of the transformation. Section 7 performs extensive experiments to verify the proposed algorithm. Section 8 concludes the paper.

2. Related works

This paper focuses on correspondence recovery. In this section, we briefly review the main methods and their relations with the proposed QPCCP scheme.

The mutual dependence between the transformation and the correspondence prompts the development of the iterative closest point (ICP) method and its variants [4–7]. In each iteration of ICP, the closest point matches are used as the correspondence and the transformation is updated based on the estimated correspondence. However, the fact that the correspondence is binary makes the optimization problem ill-posed. Without a good initialization, the ICP method may converge to a local minimum. To remedy the problem, in RPM and thin-plate-spline-RPM (TPS-RPM) methods [8–10], the correspondence is relaxed to be continuously valued which has the physical meaning of partial match. The deterministic annealing optimization technique [13] is used in order to avoid falling into many local minimums. Similar to TPS-RPM, in the proposed QPCCP method the correspondence is relaxed to be continuously valued. Different from TPS-RPM, however, no explicit annealing scheme is required to control the fuzziness of the correspondence in QPCCP.

RPM needs an annealing schedule to compute the correspondence which tends to produce a bias in aligning the centers of the mass of two point sets. To solve the problem, Sofka et al. [23] proposed a covariance driven correspondence (CDC) algorithm where the uncertainty in point correspondences is used to drive the registration process. The uncertainty is derived from the covariance matrices of the individual point locations and the covariance matrix of the estimated transformation parameters. A robust objective function and an expectation–maximization (EM) like algorithm are used to simultaneously estimate the transformation parameters, their covariance matrix and the correspondences. Neither annealing schedule nor an explicit outlier process is needed. The experimental results in [23] show that CDC has broader domain of convergence than ICP and is more robust to missing or extraneous structures in the data than RPM. However, it remains unknown how to extend this method to the case of non-rigid registration. Compared with CDC, the proposed QPCCP is general enough for non-rigid matching.

Joae Maciel et al. [24] proposed a method for the correspondence problem where most of the commonly used assumptions can be handled in a unique formulation. It implements by converting the original integer optimization problem into a continuously valued one by building a concave objective function and relaxing the search domain into its convex-hull. The special structure of the extended problem ensures its equivalence to the original one, but it can be efficiently solved by continuous optimization algorithms that avoid combinatorial search.

If discriminative features are used, the correspondence problem can be greatly alleviated. Belongie et al. [16] proposed a new feature descriptor called shape context (SC) and applied it to point matching. The correspondence problem is reduced to a binary linear assignment problem where the coefficients come from the differences of SC. However, the constraint of continuity, i.e. the neighboring points should be matched to neighboring points, is not taken into account in SC. Several schemes have been proposed to tackle the problem. In [17], SC and continuity constraint were embedded in a dynamic programming framework to detect shapes in cluttered scenes. It guarantees the optimality but imposes an ordering structure on the template point set, which limits its applicability. Relaxation labeling was used in [18] to improve the correspondence recovery result of SC by preserving the locality, which is defined by the neighborhood graphs on both point sets. In a different problem setting [19], linear programming and successive convexification were used to solve the matching problem. Such methods rely heavily on the rich amount of shape features for correspondence recovery, while the proposed QPCCP only depends on the points' positions.

POCS is a commonly used technique for constraint satisfaction by constrained projection and it has been successfully applied to signal processing, image enhancement, neural networks and optics [21,22]. Usually no closed-form solution exists for the original constrained projection problem. POCS decomposes the constraint set into several simpler subsets, for each of which the projection problem can be more easily solved. Then the projection onto the original constraint set is approximated by successive projection onto each constraint subset. Compared with the POCS technique, the QPCCP algorithm to be developed in this paper has much better performance in both accuracy and computational efficiency for point matching.

3. Problem formulation

Suppose there are two point sets to match in the d-dimensional space R^d: the template $X = \{x_i, 1 \leqslant i \leqslant n\}$ and the target $Y = \{y_j, 1 \leqslant j \leqslant m\}$. To make the point matching problem more tractable, we assume that each point in X has an analog in Y but not vice versa, which implies $n \leqslant m$. This assumption makes sense for applications such as object recognition, where the task is to deform X to match a certain portion of Y. Y may contain noise points that have no correspondences in X but each point in X has a correspondence point in Y. In other words, we do not allow model occlusion but we do allow noisy extraneous data features [11].

As shown in [8], point matching can be formulated as a combinatory optimization problem. However, it is generally difficult to solve due to the combinatory nature of point matching [8]. Therefore, in the same spirit as in [8], we pursue an approximate solution by relaxing the correspondence to be continuously valued. In doing so, the choice of the energy function becomes an important issue, because poorly formulated energy function may drive the solution away from being binary at all. We adopt here the form