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A stochastic inventory routing problem with stock-out

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ABSTRACT

In this paper, we study an inventory routing problem in which a supplier has to serve a set of retailers. For each retailer, a maximum inventory level is defined and a stochastic demand has to be satisfied over a given time horizon. An order-up-to level policy is applied to each retailer, i.e. the quantity sent to each retailer is such that its inventory level reaches the maximum level whenever the retailer is served. An inventory cost is applied to any positive inventory level, while a penalty cost is charged and the excess demand is not back-logged whenever the inventory level is negative. The problem is to determine a shipping strategy that minimizes the expected total cost, given by the sum of the expected total inventory and penalty cost at the retailers and of the expected routing cost. A hybrid roll-out algorithm is proposed for the solution of the problem and its performance is evaluated on a large set of randomly generated problem instances.

1. Introduction

Inventory routing problems (IRPs) are complex logistic problems which integrate into a unified framework the inventory management and the vehicle routing problems. IRPs typically model the resupplying policy of a set of retailers over a short or long-term planning period (time horizon), with a single product coming from a single production plant or a single warehouse, referred to as supplier. During the time horizon, the supplier plans the deliveries in terms of time, quantity and route. These situations can be observed, for instance, in the supermarket industry when the consumption of a specific product is very high over the planning horizon, so that the regular resupplying policies cannot satisfy all the customer requirements in the same period during the time horizon. This is especially true when the demands are stochastic and the vehicle capacity is limited with respect to the volume required at the retailers in order to satisfy their demands.

Since the pioneering paper by Harris (1913), several optimization models facing the integration of different types of logistic costs have been proposed. One of the most common problems is the one in which both transportation and inventory costs are taken into account. The aim is to determine shipping policies that allow to minimize the sum of these two costs. This is an interesting problem as the transportation cost raises and the inventory cost drops when shipments are performed frequently, while the contrary happens when shipments are rare over time. The integration of vehicle routing, instead than transportation, with inventory control problems has led to the development of the IRPs, that have been proposed and analyzed in different papers. Federgruen and Zipkin (1984) studied an integrated problem for managing limited resources at a plant and resupplying a set of customers within a small number of days. They defined a tactical problem to select which deliveries to execute with each set of vehicles and in which order. A stochastic demand and nonlinear inventory costs, taking into account holding and shortage costs, were considered by the authors. In the context of short planning horizons, Dror and Ball (1987) proposed the reduction of an annual distribution problem to a single period problem by introducing the definition of

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single-period costs, that reflect long-term cost, the concept of safety stock level, and clustering the customers to be serviced in a single period. The major contributions oriented to capture the long-term effects of tactical decisions were proposed in the following relevant papers. A strategical IRP with long lead times, i.e., months or even years between delivery operations, was studied by Anily and Federgruen (1990) by using a minimum number of vehicles to supply inventory. They proposed an integrated replenishment policy based on satisfying the deterministic demand of the customers and reducing the average holding and routing costs of deliveries over a long-term planning period. Bertazzi et al. (1997) studied the problem of shipping products from one origin to several destinations by taking into account several frequency-based policies with the aim of minimizing the transportation and inventory costs. Chan and Simchi-Levi (1998) analyzed the optimal trade-off strategy between inventory policy and delivery policy with the aim of minimizing the total distribution costs over an infinite horizon. They studied the asymptotic behavior of the Fixed Partition and Zero Inventory Ordering policies. Efficient approaches for long-term IRPs have been proposed by Adelman (2004), Campbell and Savelsbergh (2004), Simchi-Levi et al. (2005) and Savelsbergh and Song (2006). Few papers have been devoted to long-term IRPs with stochastic demand. Jaillet et al. (2002) study the repeated distribution of a product over a long time horizon to several customers. Several satellite facilities can be visited by the drivers to refill their vehicles. In case of stock-out, a direct delivery is made and a penalty cost is incurred. An incremental cost approximation in a rolling horizon framework is proposed to minimize the total expected delivery costs.

Kleywegt et al. (2002, 2004) formulated the stochastic IRP as a Markov decision problem (MDP) over an infinite horizon. They proposed approximated methods based on the dynamic programming in order to find good quality solutions with a reasonable computational effort. They worked with a fixed-size fleet of homogeneous vehicles, and a penalty cost taking into account the stock-out in the objective function. No split deliveries are allowed. The aforementioned contributions differ in the routing phase. In particular, the first paper (2002) deals with a stochastic IRP in which direct deliveries only are tackled, whereas the second paper (2004) studies the case in which each vehicle services up to three customers. Such routing restrictions are removed in the stochastic IRP studied by Adelman (2004). The author studied the case in which the number of customers visited in every route is unbounded and a fleet of vehicles with an unlimited number of vehicles is available. The optimal dual prices of a linear program are used in order to approximate the future costs of current actions. Heuristic approaches based on finite scenario trees, and approximating the uncertainty arising in the problem have been developed in Hvattum et al. (2009) and Hvattum and L∅kketangen (2009). In Hvattum et al. (2009), all the three stochastic IRP defined by Kleywegt et al. (2002, 2004) and Adelman (2004) are addressed. The authors proposed a hybrid approach based on the combination of the GRASP with a MIP model to solve the problems. In Hvattum and L∅kketangen (2009), the progressive hedging algorithm is examined as an alternative heuristic solution technique.

Recently, Yu et al. (in press) addressed a stochastic IRP with split delivery. The authors introduced different service levels at the warehouses and customers with the aim of taking into account the stock-out. The IRP studied by the authors is a stochastic version of the deterministic one proposed by Yu et al. (2008), and devoted to solve large scale IRPs. The stochastic problem is modeled as an approximate stochastic IRP exploiting the transformation of stochastic components in deterministic ones, and near-optimally solved by using an hybrid approach based on a Lagrangian relaxation followed by local search techniques.

For an in-depth overview of this area of research, the reader is referred to Federgruen and Simchi-Levi (1995), Campbell et al. (1998), Cordeau et al. (2007), Moin and Salhi (2007), Bertazzi et al. (2008) and Andersson et al. (2010).

This paper is focused on an IRP with stochastic demand, where stock-outs may occur during the time horizon. These situations can be observed, for instance, in the supermarket industry when the consumption of a specific product is quite high so that the regular resupplying policy is not able to satisfy all the customer requirements in the same period during the time horizon. This is especially true when the demands are stochastic and the vehicle capacity is quite limited with respect to the volume required at the retailers. Each retailer defines a maximum inventory level. An order-up-to level policy is applied, i.e. the quantity sent to each retailer is such that its inventory level reaches the maximum level whenever the retailer is served. This policy has been introduced for the case with deterministic demand by Bertazzi et al. (2002), is relevant from a practical point of view and represents a good starting point to study such a complex problem. An inventory cost occurs if the inventory level is positive. Instead, whenever the inventory level is negative, a penalty cost comes into play and the excess demand is not backlogged. Shipments are performed by a fleet composed of one vehicle with given transportation capacity. The problem is to determine a shipping strategy that minimizes the expected total cost, given by the sum of the expected total inventory and penalty costs at the retailers and of the expected routing cost over the time horizon.

We first provide a dynamic programming formulation of the problem that allows us to design a hybrid rollout algorithm aimed at finding good quality solutions of the problem. Rollout algorithms are a class of heuristic algorithms that can be used to solve deterministic and stochastic dynamic programming problems. The basic idea is to use the cost obtained by applying a heuristic, called base policy, to approximate the value of the optimal cost-to-go in a one-step lookahead policy. These algorithms are very appealing from the practical point of view, as they are easy to be implemented and guarantee a no worse, and usually much better, performance than the corresponding base policy. These algorithms have been originally proposed in the context of Neuro-Dynamic Programming/Reinforcement Learning (see Bertsekas and Tsitsiklis, 1996; Tesauro and Galperin, 1997; and Sutton and Barto, 1998). They have been applied to stochastic scheduling by Bertsekas and Castanõn (1998) and to vehicle routing problems with stochastic demand by Secomandi (2000, 2001, 2003). Our rollout algorithm is hybrid in the sense that the approximate cost-to-go is obtained by exactly solving a mixed integer linear programming model, that is the deterministic counterpart of our problem in which the future demand is set equal to the average demand. This deterministic problem is a generalization of the inventory routing problem with order-up-to level policy (VMIR-OU) studied in Archetti et al. (2007), as no stock-out at the retailers is allowed in that paper. The problem with stock-out is particularly relevant