Image and Vision Computing 27 (2009) 756–770

Contents lists available at ScienceDirect

Image and Vision Computing

journal homepage: www.elsevier.com/locate/imavis

Automatic noise estimation in images using local statistics. Additive and multiplicative cases

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ARTICLE INFO

Article history:
Received 8 May 2007
Received in revised form 11 January 2008
Accepted 4 August 2008

Keywords:
Noise estimation
Mode
Restoration
Gaussian noise
Local statistics

ABSTRACT

In this paper, we focus on the problem of automatic noise parameter estimation for additive and multiplicative models and propose a simple and novel method to this end. Specifically we show that if the image to work with has a sufficiently great amount of low-variability areas (which turns out to be a typical feature in most images), the variance of noise (if additive) can be estimated as the mode of the distribution of local variances in the image and the coefficient of variation of noise (if multiplicative) can be estimated as the mode of the distribution of local estimates of the coefficient of variation. Additionally, a model for the sample variance distribution for an image plus noise is proposed and studied. Experiments show the goodness of the proposed method, specially in recursive or iterative filtering methods.

1. Introduction

Noise estimation is a task of paramount importance in most image restoration techniques. These techniques are usually based on a degradation model where a noise-dependent-parameter is to be estimated and it controls the amount of filtering to be executed. For instance, the Wiener filter (in any of its multiple versions [1]) is built on the assumption that the image is corrupted by additive random noise. Thus the parameter to be estimated is the noise power spectrum or the variance of noise. If a Lee's filter [2] for multiplicative noise is considered, then the coefficient of variation (CV) of noise (i.e. the ratio of the standard deviation and the mean) is the one needed. Regardless of the theoretical goodness of the different methods to restore an image, the key point is the estimation of such noise parameters.

Noise estimation techniques in the spatial domain have been classified as block based and filtering based methods [3]. The former deals with the local standard deviation of the image, which is calculated using $M \times N$ blocks. In the latter, the image is filtered by a low-pass filter and the noise is estimated using the standard deviation of the difference between the original and the filtered images. A number of involved variations based on these approaches have been reported, such as methods based on wavelets [4,5], singular-value-decomposition [6], fuzzy logic [7] or median absolute deviation [8]. Alternatively, fast and simple solutions have also been reported [1]; for instance, in order to estimate the variance of additive noise, the minimum or the average of the locally estimated variances are considered, even though it is known that the former underestimates while the latter overestimates the pursued value [9,10]. In other fields this problem has also been tackled; specifically, in the context of speech processing some methods have being developed for time-varying background noise estimation, as the ones analyzed by [11,12].

In this paper, we present a novel approach based on local sample statistics, specifically on the distribution of such statistics. We will show that noise parameters may be accurately estimated using the mode of the population of local estimations. The mode is a fairly good estimator of the variance (if additive noise) or the CV (if multiplicative) of noise when the image to work with has a sufficiently great amount of low-variability areas so as to make the local hypothesis of "constant plus noise" acceptable; fortunately, this is not too a restrictive hypothesis for many real world (untextured) images. We will also show that an accurate estimation of noise is even more important when dealing with iterative or recursive filtering schemes, in which the level of noise varies from one iteration to another.

The paper is organized as follows: In Section 2 the additive and multiplicative models of noise used in this paper are introduced. Section 3 discusses the theory of the approach, Section 4 presents results and compares the methodology to other estimation procedures. Conclusions are summarized in Section 5. Some appendices have been added to ease the paper readability.

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doi:10.1016/j.imavis.2008.08.002

2. Noise models and estimation

In this paper we will focus on images degraded by either additive or multiplicative noise. For the former we will consider the following model

$$g_{i,j} = f_{i,j} + n_{i,j} \tag{1}$$

where $f_{i,j}$ is the intensity at pixel (i,j) in the original image, $g_{i,j}$ is the intensity at pixel (i,j) in the degraded image and $n_{i,j}$ is a Gaussian noise sequence with zero mean and $\mathrm{Var}(n_{i,j}) = \sigma_n^2$ constant throughout the image. Assuming that the noise and the original image are independent, the variance of the degraded image can be written [1]

$$\sigma_{g_{i,j}}^2 = \sigma_{f_{i,j}}^2 + \sigma_n^2 \tag{2}$$

being both $\sigma_{g_{i,j}}^2$ and $\sigma_{f_{i,j}}^2$ local variances.

For the multiplicative noise the following the model will be used

$$g_{i,j} = f_{i,j} u_{i,j} \tag{3}$$

where $u_{i,j}$ is the multiplicative noise with mean and variance constant throughout the image and respectively denoted by $E[u_{i,j}] = \bar{u}$ and $\mathrm{Var}(u_{i,j}) = \sigma_u^2$. The (square) coefficient of variation (CV) of noise will be

$$C_u^2 = \frac{\sigma_u^2}{\bar{u}^2} \tag{4}$$

and it is also constant throughout the image. The local (square) CV of the degraded image is then defined as

$$C_{i,j}^2 = \frac{\sigma_{g_{i,j}}^2}{\bar{g}_{i,j}^2} \tag{5}$$

where $\bar{g}_{i,j}$ is the local mean of the degraded image. This equation can be rewritten as [13,14]

$$C_{i,j}^2 = \frac{\sigma_{f_{i,j}}^2}{\bar{f}_{i,j}^2}(1 + C_u^2) + C_u^2 \tag{6}$$

According to Eq. (2), if at some region the equality $\sigma_{f_{i,j}}^2 = 0$ holds, i.e., the image is locally constant (plus noise), then it is clear that

$$\sigma_{g_{i,j}}^2 = \sigma_n^2 \tag{7}$$

Therefore, within a uniform area (in terms of the signal content) the variance of the degraded image equals the variance of noise. The same reasoning can be applied to Eq. (6) for multiplicative noise, i.e., within a homogeneous area

$$C_{i,j}^2 = C_u^2 \tag{8}$$

Although this reasoning is well-known, some consequences should be taken into account (we will now focus on the additive case; the reasoning for the multiplicative case is similar, by replacing the variance with the CV): specifically, according to the previous statement, one straightforward way to estimate σ_n^2 is to calculate the variance within homogeneous regions [15,16], where the variance of the original image is close to zero, so Eqs. (7) and (8) hold. This method has the drawback of requiring either some sort of automatic procedure or manual selection to detect these homogeneous regions. It is also highly sensitive to errors, outliers and inhomogeneities.

Alternatively, if estimations were totally accurate, and according to Eq. (2), then σ_n^2 should be

$$\hat{\sigma}_n^2 = \sigma_{\min}^2 = \min_{i,j}\{\sigma_{g_{i,j}}^2\} \tag{9}$$

However, this estimate is, in practice, biased towards zero due to the sensitivity of the min operator to outliers and the real distribution of the local variance estimator itself; as a result it frequently underestimates the real value of σ_n^2. Some authors use the average instead

$$\hat{\sigma}_n^2 = \sigma_{\mathrm{ave}}^2 = \frac{1}{N}\sum_{i,j}\sigma_{g_{i,j}}^2 \tag{10}$$

(with N the number of points in the summation) but this method tends, conversely, to overestimate.

A walk-around has been proposed elsewhere [9], where some intermediate value is calculated by introducing a free parameter λ ranging within the interval (0,1). The estimator is then

$$\hat{\sigma}_n^2 = \lambda\sigma_{\min}^2 + (1-\lambda)\sigma_{\mathrm{ave}}^2. \tag{11}$$

Another common noise estimator in video and speech processing is [4]

$$\hat{\sigma}_n = \sigma_{\mathrm{MAD}} = 1.4826 \times \mathrm{MAD}(y_{i,j}^H) \tag{12}$$

which assumes the noise standard deviation as proportional to the median absolute deviation (MAD) of the wavelet coefficients in the highest frequency subband, $y_{i,j}^H$. The MAD is defined (for some dataset g_i)

$$\mathrm{MAD} = \mathrm{median}_i(||g_i - \mathrm{median}_k(g_k)||)$$

In [8], the authors propose a similar estimator for the CV

$$\widehat{C_u^2} = \frac{1.4826}{\sqrt{2}} \times \mathrm{MAD}(\nabla \log g_{i,j}) \tag{13}$$

As it has been stated in the introduction, more complex methods have also been reported in order to estimate noise statistics. These methods are not only based on the local variance $\sigma_{g_{i,j}}^2$ but on other parameters of the image. See for example [3,5–7,17,18]. In this paper a novel simple procedure is introduced.

3. Noise estimation

Our aim is to find a fast, simple and reliable method to estimate σ_n^2 when additive Gaussian Noise is considered or C_u^2 for the multiplicative case. Let us take a previous practical view of the problem of estimation. We will focus first on the case of additive noise. According to Eq. (2) the theoretical effect of adding Gaussian noise to an image is that its local variance increases an amount σ_n^2. In Fig. 1 we show the reference image that we will use for this experiment, and the same image with additive Gaussian noise with zero mean and $\sigma_n = 20$ (for an image in the range [0,255]).

The (normalized) distribution of the local variance of the image is estimated and shown in Fig. 2a. According to Eq. (2) one would expect that the distribution of the noise image were just this distribution shifted to the right an amount σ_n^2, as shown in Fig. 2b. As previously stated, the straightforward estimator would be the minimum of the distribution, as in Eq. (9). However, as the sampling estimation of the local variance is used, this estimator will also have a variance itself [19], and its distribution differs from that

(a) Original image (b) Noisy image

Fig. 1. Real image used for the experiments. (a) Original image (256 gray levels). (b) Image with Gaussian noise with 0 mean and $\sigma_n = 20$.