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Median-based image thresholding ☆

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ABSTRACT

In order to select an optimal threshold for image thresholding that is relatively robust to the presence of skew and heavy-tailed class-conditional distributions, we propose two median-based approaches: one is an extension of Otsu's method and the other is an extension of Kittler and Illingworth's minimum error thresholding. We provide theoretical interpretation of the new approaches, based on mixtures of Laplace distributions. The two extensions preserve the methodological simplicity and computational efficiency of their original methods, and in general can achieve more robust performance when the data for either class is skew and heavy-tailed. We also discuss some limitations of the new approaches.

1. Introduction

Image thresholding aims to partition an image into K predetermined, mutually-exclusive classes, C_1, …, C_K, based on $K-1$ intensity thresholds. Most commonly, $K=2$ and the image is partitioned into the background and the foreground. As an initial procedure for realising image segmentation, thresholding has a long history of investigation, motivated by a broad range of practical applications of image analysis and object recognition. Comprehensive overviews and comparative studies of image thresholding can be found in [15,4,17,16], for example.

Many, and the most-widely used, approaches to image thresholding are based on analysis of the histogram of intensities in an image, searching for an optimal threshold t^* to divide the histogram into two parts, C_1 with intensities lower than t^* and C_2 for the remainder.

Among these approaches, two of the most popular are Otsu's method [12] and Kittler and Illingworth's minimum-error-thresholding (MET) method [8]. Otsu's method is adopted as the method for automatic image thresholding in some free and commercial software, such as GIMP (www.gimp.org) and MATLAB (The MathWorks, Inc.). The MET method is ranked as the best in a comprehensive survey of image thresholding conducted by [16].

In image thresholding, determination of an optimal threshold t^* is often based on the estimation of measures of location and dispersion of intensities in C_1 and C_2. As with many other approaches, both Otsu's method and the MET method use the sample mean and the sample standard deviation to estimate location and dispersion, respectively.

It is well known that, when the distribution for class C_k is skew or heavy-tailed, or when there are outliers in the sample from C_k, the median is a more robust estimator of location than the mean. When the median is chosen for location, the mean absolute deviation from the median (denoted by MAD) is usually chosen as the estimator of dispersion.

Therefore, in order to select a t^* that is more robust to the presence of skew and heavy-tailed distributions for C_k than those selected by Otsu's method and the MET method, we propose in section 2 two median-based approaches to image thresholding. One of them is an extension of Otsu's method and the other is an extension of the MET method; both methods are based on the use of the MAD. Like their original versions, the two new approaches remain methodologically simple and computationally efficient.

The relationship between Otsu's method and the MET method has been investigated by [9,21], among others. [9] shows that both methods can be derived from maximisation of log-likelihoods based on mixtures of Gaussian distributions. In section 3, we present theoretical interpretation of their median-based extensions from the perspective of the maximisation of log-likelihoods for mixtures of Laplace distributions.

Some limitations of the median-based approaches are discussed in section 4 and a summary is made in section 5.

2. Methodology

Each of the N pixels in an image χ is represented by its intensity x_i, $i=1,\ldots,N$. A threshold t partitions the image into two classes $C_1(t)$ and $C_2(t)$, where $C_1(t)=\{i\colon 0\le x_i\le t,\ 1\le i\le N\}$ and $C_2(t)=\{i\colon t<x_i\le T,$

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$1 \le i \le N\}$, in which T is the largest possible intensity, which is 255 for an 8-bit grey-level image (i.e. $x_i \in [0, T]$).

The histogram for the image χ, denoted by $\{h(x)\}$, can be constructed by counting the frequencies of the intensities and dividing them by N, such that $\sum_{x=0}^{T} h(x) = 1$.

2.1. Otsu's method and its median-based extension

2.1.1. Otsu's method

Otsu's rule [12,9] for defining the optimal threshold t can be written as

$$t_O^* = \underset{t}{\operatorname{argmin}} J_O(t) = \underset{t}{\operatorname{argmin}} \left\{ \omega_1(t) s_1^2(t) + \omega_2(t) s_2^2(t) \right\}, \tag{1}$$

where $\omega_1(t)$ and $\omega_2(t)$ are the proportions of pixels representing classes $C_1(t)$ and $C_2(t)$ determined by a threshold t, $s_1(t)$ and $s_2(t)$ are the (biased) sample standard deviations for $C_1(t)$ and $C_2(t)$, respectively, defined as

$$\omega_1(t) = \sum_{x=0}^{t} h(x), \ \omega_2(t) = \sum_{x=t+1}^{T} h(x) = 1 - \omega_1(t), \tag{2}$$

$$s_1^2(t) = \sum_{x=0}^{t} \left[\frac{h(x)}{\omega_1(t)} \{x - \bar{x}_1(t)\}^2 \right], \ s_2^2(t) = \sum_{x=t+1}^{T} \left[\frac{h(x)}{\omega_2(t)} \{x - \bar{x}_2(t)\}^2 \right], \tag{3}$$

in which $\bar{x}_1(t) = \sum_{x=0}^{t} \{x h(x) / \omega_1(t)\}$ and $\bar{x}_2(t) = \sum_{x=t+1}^{T} \{x h(x) / \omega_2(t)\}$ are the sample means for $C_1(t)$ and $C_2(t)$, respectively.

2.1.2. A median-based extension

As mentioned in section 1, we envisage that the use of the median instead of the mean may provide a t that is more robust to the presence of skew and heavy-tailed distributions for C_k than those selected by Otsu's method and the MET method. Therefore, a median-based extension of Otsu's method, derived in a natural way by substituting the MAD for s^2 (not for s for theoretical reasons explained in section 3), provides a rule for selecting t (denoted by g for distinctive purposes hereafter) as follows:

$$g_O^* = \underset{t}{\operatorname{argmin}} J_O^M(t) = \underset{t}{\operatorname{argmin}} \{\omega_1(t) MAD_1(t) + \omega_2(t) MAD_2(t)\}, \tag{4}$$

where $\text{MAD}_k(t)$, the mean absolute deviation from the median for class $C_k(t)$, is given, for $k = 1, 2$, by

$$MAD_1(t) = \sum_{x=0}^{t} \left\{ \frac{h(x)}{\omega_1(t)} |x - m_1(t)| \right\}, \tag{5}$$

$$MAD_2(t) = \sum_{x=t+1}^{T} \left\{ \frac{h(x)}{\omega_2(t)} |x - m_2(t)| \right\}, \tag{6}$$

in which $m_1(t) = \text{med}\{x_i: i \in C_1(t)\}$ and $m_2(t) = \text{med}\{x_i: i \in C_2(t)\}$ are the sample medians for $C_1(t)$ and $C_2(t)$, respectively.

2.1.3. Multi-level thresholding

When there are more than two classes predetermined for an image (i.e. $K > 2$), it would be better to use more than one threshold to partition the image into these classes, leading to a multi-level thresholding problem.

For multi-level thresholding, Otsu's rule for selecting optimal thresholds $\mathbf{t}^* = (t_1^*, \ldots, t_{K-1}^*)$ can be written as

$$t_O^* = \underset{\mathbf{t}}{\operatorname{argmin}} \sum_{k=1}^{K} \left\{ \omega_k(\mathbf{t}) s_k^2(\mathbf{t}) \right\}, \tag{7}$$

where, similarly to the version in section 1, $\omega_k(\mathbf{t})$ and $s_k^2(\mathbf{t})$ are defined for $C_k(\mathbf{t})$.

Therefore, for multi-level thresholding, the rule underlying Otsu's median-based extension becomes

$$g_O^* = \underset{\mathbf{t}}{\operatorname{argmin}} \sum_{k=1}^{K} \{\omega_k(\mathbf{t}) MAD_k(\mathbf{t})\}. \tag{8}$$

2.2. The MET method and its median-based extension

2.2.1. The MET method

The MET method [8] selects t^* as

$$t_M^* = \underset{t}{\operatorname{argmin}} J_M(t) = \underset{t}{\operatorname{argmin}} \left\{ \omega_1(t) log \frac{s_1(t)}{\omega_1(t)} + \omega_2(t) log \frac{s_2(t)}{\omega_2(t)} \right\}, \tag{9}$$

where $\omega_1(t)$, $\omega_2(t)$, $s_1(t)$ and $s_2(t)$, defined in Eqs. (2) and (3), are positive here.

2.2.2. A median-based extension

By analogy with section 1, the rule underlying a median-based extension of the MET method can be derived by substituting the MAD for s (not as with section 2 for s^2 for theoretical reasons explained in section 3) as

$$g_M^* = \underset{t}{\operatorname{argmin}} J_M^M(t) = \underset{t}{\operatorname{argmin}} \left\{ \omega_1(t) log \frac{MAD_1(t)}{\omega_1(t)} + \omega_2(t) log \frac{MAD_2(t)}{\omega_2(t)} \right\}. \tag{10}$$

2.2.3. Multi-level thresholding

The multi-level-thresholding versions of the MET method and its median-based extension are readily given by

$$\mathbf{t}_M^* = \underset{\mathbf{t}}{\operatorname{argmin}} \sum_{k=1}^{K} \left\{ \omega_k(\mathbf{t}) log \frac{s_k(\mathbf{t})}{\omega_k(\mathbf{t})} \right\}, \tag{11}$$

$$g_M^* = \underset{\mathbf{t}}{\operatorname{argmin}} \sum_{k=1}^{K} \left\{ \omega_k(\mathbf{t}) log \frac{MAD_k(\mathbf{t})}{\omega_k(\mathbf{t})} \right\}. \tag{12}$$

3. Theoretical interpretation

3.1. Relationship with Laplace mixtures

A straightforward and intuitive interpretation of Otsu's rule, as shown in Eq. (1), is that it aims to minimise the within-classes variance $J_O(t)$, a measure of dispersion, of the intensity. Correspondingly, an interpretation of the median-based extension of Otsu's method, as shown in Eq. (4), is that the extension aims to minimise the within-classes mean absolute deviation from the median $J_O^M(t)$, another measure of dispersion, of the intensity.

Alternatively and insightfully, as mentioned in section 1, [9] shows that both Otsu's method and the MET method can be derived from maximisation of log-likelihoods based on mixtures of Gaussian distributions. The same type of interpretation can be found in [8] from the derivation of the MET method, although it was not explicitly mentioned there. Analogously to that, we present a similar theoretical interpretation of the median-based approaches, from the perspective of the maximisation of log-likelihoods based on mixtures of Laplace distributions.

Suppose that the intensity of class C_k follows a Laplace distribution, of which the probability density function $p(x|C_k)$ is defined as

$$p(x|C_k) = \frac{1}{2\beta_k} exp\left(-\frac{|x - \alpha_k|}{\beta_k} \right), \tag{13}$$

where α_k is a location parameter and β_k is a positive scale parameter. The maximum likelihood estimator (MLE) of α_k is the sample median