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Algorithms for fast computation of Zernike moments and their numerical stability ☆

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ABSTRACT

Accuracy, speed and numerical stability are among the major factors restricting the use of Zernike moments (ZMs) in numerous commercial applications where they are a tool of significant utility. Often these factors are conflicting in nature. The direct formulation of ZMs is prone to numerical integration error while in the recent past many fast algorithms are developed for its computation. On the other hand, the relationship between geometric moments (GMs) and ZMs reduces numerical integration error but it is observed to be computation intensive. We propose fast algorithms for both the formulations. In the proposed method, the order of time complexity for GMs-to-ZMs formulation is reduced and further enhancement in speed is achieved by using quasi-symmetry property of GMs. The existing q-recursive method for direct formulation is further modified by incorporating the recursive steps for the computation of trigonometric functions. We also observe that q-recursive method provides numerical stability caused by finite precision arithmetic at high orders of moment which is hitherto not reported in the literature. Experimental results on images of different sizes support our claim.

1. Introduction

Zernike moments (ZMs) find wide applications in pattern recognition [1,2], content-based image retrieval [3,4], optical character recognition [5–8], image reconstruction [9,10], edge detection [11,12], image watermarking [13,14] and palmprint verification [15]. Among the many moment shape descriptors, ZMs are observed to be the most desirable ones for shape description [3] and because of its superior performance, ZMs descriptor has been accepted by MPEG-7 as a region based shape descriptor [16]. The superiority of ZMs stems from the fact that their basis functions are orthogonal thereby eliminating the redundancy in information content as compared to other orthogonal radial moments such as pseudo Zernike and orthogonal Fourier–Mellin moments. They are simple to compute and the absolute values of moments are invariant to rotation. Translation invariance is achieved by computing moments about the centroid of image and scale invariance is obtained by either normalizing the image [10] or by using a modified form of ZMs [17]. Theoretically, these moments are rotation invariant but their discrete implementations exhibit unacceptable variations in moment values and recently this issue has been solved to a large extent by computing the moments in polar coordinates [18] and by interpolating the moments in moment space [19].

ZMs are implemented in discrete space by approximating the integration with the summation. The polynomial functions involved in the basis functions are evaluated at discrete points where the image function is defined which is assumed to be constant over a pixel grid. If we compute moment of all orders up to a maximum order L for an image of the size $N \times N$ pixels, then the order of time complexity turns out to be $O(N^2L^3)$. This is a large order when both N and L are large. This is obvious from the fact that there are N^2 image points at which the polynomial functions are evaluated. The total number of ZMs is $\frac{1}{2}(L+1)(L+2)$. The average number of coefficients of the polynomials is $\frac{1}{4}(L+4)$ or $\frac{1}{4}(L+3)$ for L even or odd. Thus the overall time complexity is $O(N^2L^3)$. Further slowdown in speed is caused by the presence of factorial terms in the coefficients of the polynomials, power of the radial term i.e. r^k, $k=0,1,\ldots,L$, and the evaluation of trigonometric functions, $\cos(m\theta)$ and $\sin(m\theta)$, $m=0,1,\ldots,L$. Chong et al. [20] carry out an extensive survey of fast methods and propose a new method which is popularly known as q-recursive method, where q represents the repetition term in ZMs. The q-recursive method is reported to be the best method among all recursive methods. Our numerical experiments also support the claim of [20]. Further enhancements in speed are achieved by exploiting the property of symmetry in the calculation of radial polynomials and trigonometric functions [17,21,22]. Although symmetry property is used for trigonometric functions, their evaluation even on an octant of the unit disc is a time consuming process. The focus of some of the best methods reported in [20] is to reduce the time complexity of computing the polynomial terms. Notably, the time complexity of computing the polynomial terms is reduced from $O(L)$ to $O(1)$, where

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$O(1)$ is independent of L. The factorial terms involved in the computation of the coefficients of the polynomials are also eliminated. This yields a significant improvement in speed over the previous fast methods.

ZMs suffer from two major errors-numerical error and geometric error [23]. Numerical error consists of two parts. The contribution to the first part of the error arises because of discretization/quantization of the image function which is inherent in digital images. The second part, called numerical integration error, is caused when the double integration is approximated by double summations and the basis functions are evaluated at the center of a grid. An approach suggested by Liao and Pawlak [23] is to use numerical integration for better approximation. This approach, however, slows down the computation significantly. Recently, the exact computation of ZMs are proposed by Kotoulas and Andreadis [24] and by Wee and Paramesran [25]. A mathematical relationship is used to derive ZMs from geometric moments(GMs). Since GMs are computed more accurately in a square domain by assuming constant values of image function over pixel grids, the relationship provides a better approximation for the ZMs which reduces numerical error. The effect due to geometric error is reduced by considering a circle which encloses the image completely [25]. Another approach to eliminate geometric error is proposed by Xin, Pawlak and Liao [18] which uses a circular type of image tiling in polar coordinates. The price paid for that is the presence of interpolation error, fortunately being smaller order than the geometric one.

A new real time hardware architecture for the computation of ZMs using digital filters is presented which outperforms existing software approaches especially for large images, allowing real time processing of images up to 4 megapixels [24]. The existing software methods of GMs-to-ZMs calculation have a time complexity of $O(N^2L^5)$. However, when intermediate results are saved in look up tables, the time complexity reduces to $O(L^2 \times max(N^2,L^3))$. In this paper, we propose a new method which reduces time complexity from $O(L^2 \times max(N^2,L^3))$ to $O(L^2 \times max(N^2,L^2))$. The proposed time complexity is comparable to the fast recursive methods whose time complexity is $O(N^2L^2)$. It will be shown that the proposed method provides the fastest method for ZMs calculation when GMs are used.

Another major issue involved with the ZMs calculation pertains to the numerical instability due to finite precision arithmetic when the order of moments is large. The non-recursive formulation of ZMs or the computation of ZMs through GMs suffers from numerical instability for high orders of moments(>45) and instability is reflected through the increasing trend of image reconstruction error starting from moment order 45. A detailed comparative study of numerical stability of recursive algorithms is performed by Papakostas et al. [26,27]. Two types of error-overflow and finite precision errors-are taken into account while analyzing their effects on numerical behavior of radial polynomials and the coefficients associated with the recursive steps. It is observed that the finite precision error is of major concern, since a possible error in a step of the algorithm may be accumulated iteration by iteration, resulting to unreliable quantities. Among the various recursive algorithms, the q-recursive method is found to be more stable than the other methods. The authors, however, have not shown the effect of numerical errors on the accuracy of ZMs which is reflected through image reconstruction error for high orders of moment. The major sources of numerical instability are attributed with the high order factorial terms and high order of radial polynomials. An arbitrary precision arithmetic is used to accurately compute ZMs at high orders [28], albeit at the cost of high computation time. The authors have, however, shown results for moment orders up to 70. In an attempt to compute ZMs accurately and preserve rotation invariance, Xin et al. [18] compute ZMs in polar coordinates and show that their method provides numerical stability at high orders of moment. This method is suitable for inherently circular images, e.g., images appearing in ophthalmology. During the course of the implementation of several algorithms, we observed that the q-recursive method [20] is quite stable and its stability increases with the size of image. Our observations are in line with the findings reported by [26,27]. For an image of the size 64×64 pixels it provides numerical stability for moment orders up to 180, and for 512×512 pixels image the stability is exhibited even for moment orders up to 450. Thus q-recursive method is not only one of the fastest methods but stable too. We also observe that the q-recursive method is faster than the method based on the relationship between GMs and ZMs for large orders of moment. We further enhance its speed by developing recurrence relations for the evaluation of trigonometric functions.

The above discussions and observations set the directions for our proposed work as follows:

(i) Numerical error in ZMs is reduced by using the relationship of GMs and ZMs. The computation cost of existing methods based on this relationship is very high. No attempt has previously been made to derive fast algorithm for ZMs based on this approach. We propose a fast method for the same.

(ii) Fast method such as q-recursive method with symmetry/quasi-symmetry property exists for the direct formulation of ZMs. However, it suffers from numerical error. Apart from being fast, we discuss its numerical stability at high orders of moment. In addition, we propose a faster method for ZMs by developing recursive relations for trigonometric functions. We call this method qθ-recursive method.

The paper does not address the issues of discretization/quantization error and robustness to noise.

The rest of the paper is organized as follows. Section 2 presents an overview of ZMs formulation for the direct approach and for the approach based on a mathematical relationship between GMs and ZMs. Section 3 deals with the proposed fast algorithms based on the two approaches. Pseudo codes are also presented which facilitate the understanding of the algorithms. Numerical experimental results are elaborated in Section 4 followed by conclusions in Section 5.

2. Zernike moments (ZMs)

2.1. Radial polynomial (direct) form of ZMs

ZMs of order n and repetition m (m and q are used interchangeably) of an image function $f(x,y)$ in two dimensions over a unit disc are characterized by the equation:

$$Z_{nm} = \frac{n+1}{\pi} \iint_{x^2+y^2<=1} f(x,y) V^*_{nm}(x,y) dxdy \tag{1}$$

where the image function is defined over a square domain $N \times N$ and $V^*_{nm}(x,y)$ is the complex conjugate of the complex polynomials $V_{nm}(x,y)$ given by

$$V_{nm}(x,y) = R_{nm}(r) e^{jm\theta} \tag{2}$$

where, $r = \sqrt{x^2+y^2}, 0 \le r \le 1, j = \sqrt{-1}, n \ge 0, |m| \le n, n-m = even$, and

$$\theta = tan^{-1}\left(\frac{y}{x}\right) \tag{3}$$

The angle θ is between 0 and 2π and is measured w.r.t. x-axis in counter clockwise direction. The Zernike real valued radial polynomials $R_{nm}(r)$ are given by

$$R_{nm}(r) = \sum_{\substack{k=|m|,\\ n-k=even}}^{n} B_{nmk} r^k \tag{4}$$