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3D block-based medial axis transform and chessboard distance transform based on dominance

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ABSTRACT

Traditionally, the block-based medial axis transform (BB-MAT) and the chessboard distance transform (CDT) were usually viewed as two completely different image computation problems, especially for three dimensional (3D) space. In fact, there exist some equivalent properties between them. The relationship between both of them is first derived and proved in this paper. One of the significant properties is that CDT for 3D binary image V is equal to BB-MAT for image V' where it denotes the inverse image of V. In a parallel algorithm, a cost is defined as the product of the time complexity and the number of processors used. The main contribution of this work is to reduce the costs of 3D BB-MAT and 3D CDT problems proposed by Wang [65]. Based on the reverse-dominance technique which is redefined from dominance concept, we achieve the computation of the 3D CDT problem by implementing the 3D BB-MAT algorithm first. For a 3D binary image of size N^3, our parallel algorithm can be run in $O(\log N)$ time using N^3 processors on the concurrent read exclusive write (CREW) parallel random access machine (PRAM) model to solve both 3D BB-MAT and 3D CDT problems, respectively. The presented results for the cost are reduced in comparison with those of Wang's. To the best of our knowledge, this work is the lowest costs for the 3D BB-MAT and 3D CDT algorithms known. In parallel algorithms, the running time can be divided into computation time and communication time. The experimental results of the running, communication and computation times for the different problem sizes are implemented in an HP Superdome with SMP/CC-NUMA (symmetric multiprocessor/cache coherent non-uniform memory access) architecture. We conclude that the parallel computer (i.e., SMP/CC-NUMA architecture or cluster system) is more suitable for solving problems with a large amount of input size.

1. Introduction

Consider a two dimensional (resp. three dimensional) binary image consisting of foreground (i.e., black or 1) pixels (resp. voxels) and background (i.e., white or 0) pixels (resp. voxels). The extraction of the information about the shape and the position of the foreground pixels relative to each other is frequently used in the fields of image processing and computer vision. (Remarks: Without loss of generality, in this paper, we say that a point is represented as a pixel in a 2D plane or a voxel in a 3D space.) This can be done by two common techniques. One is the distance transform (DT) introduced by Rosenfeld and Pfaltz [54]. The other is the medial axis transform (MAT) originally explored by Blum [3].

1.1. The concept of DT

The DT is an operation that converts a binary image to an image, where each pixel has a value corresponding to the distance to the nearest foreground pixel. The chessboard distance transform (CDT) and the Euclidean distance transform (EDT) are both DTs based on the chessboard distance metrics and the Euclidean distance metrics, respectively. Given an $N \times N$ binary image M with $m(i, j) \in \{0,1\}$, $0 \le i, j \le N-1$, where 1 (resp. 0) denotes a 1-pixel or foreground (resp. 0-pixel or background) pixel. Let $B_1 = \{b_1(i,j) \mid m(i,j) = 1 \text{ of } M\}$ be the set of all

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foreground pixels of M, and $B_0 = \{ b_0(i,j) \mid m(i,j) = 0$ of $M\}$ be the set of all background pixels of M.

Then the 2D EDT of an image M can be computed by

$$\mathrm{EDT}(m(i,j)) = \min_{(x,y)\in B_1} \left((i-x)^2 + (j-y)^2\right)^{1/2}, \text{ for } 0 \le i,j \le N-1.$$

The two dimensional distance function of L_k metric of the plane by d_k as defined in [57] is listed as follows:

$$\mathrm{d}_k((i,j),(x,y)) = \left(|i-x|^k + |j-y|^k\right)^{1/k} \text{ where } 1 \le k < \infty,$$
$$\mathrm{d}_\infty((i,j),(x,y)) = \max(|i-x|,|j-y|).$$

d_1 is called the “city block distance”, d_2 is called the “Euclidean distance” and d_∞ is called the “chessboard distance”. Then the 2D CDT of an image M is to find an array

$$\mathrm{CDT}(m(i,j)) = \min_{(x,y)\in B_1} \{\max(|i-x|, |j-y|)\}, \text{for } 0 \le i,j \le N-1. \quad (1-1)$$

In Eq. (1-1), in other words, the CDT($m(i, j)$) is the array for each 0-pixel of B_0 and 1-pixel of B_1 to find its nearest 1-pixel of B_1 in the binary image M. It is clear that the value of CDT($m(i, j)$) is equal to 0 for each 1-pixel of B_1 to find its nearest 1-pixel of B_1 in the binary image M.

1.2. The concept of MAT

Note that the medial axis (MA) is also called medial axis transform (MAT), symmetry axis transform (SAT), skeleton, grassfire transform, axial symmetries, and etc. The MAT has been studied extensively so far [10,12,19,23,28,34,36,56]. There are many medial axis (MA) definitions and computations such as (1) Locus of points equidistant from contour by the symmetry set (SS) [7] and bisectors [49], (2) Grass–fire model [8,40], (3) Locus of centers of maximal circles [54,55], (4) Local maxima in distance transform [2,32], (5) Thinning [31,44,47,48,60], and (6) The shock graph [29] defined as special points of the flow along the MA. There are two methods for the MAT described by Ferreira and Ubéda [20]. One is the distance-based MAT (denoted as DB-MAT) [54,55], which is defined as a recovering of the object by maximal digital disks (resp. spheres) of 1-pixels (resp. 1-voxels) in a 2D (resp. 3D) space included in the object. A maximal digital disk (resp. sphere) is a digital disk (resp. sphere) that is not contained in any other digital disk (resp. sphere). The second method is the block-based MAT (denoted as BB-MAT) [45], which is a recovering of the object by maximal square (resp. cube) blocks of 1-pixels (resp. 1-voxels) in a 2D (resp. 3D) space. A maximal square (resp. cube) block is a square (resp. cube) block that is not contained in any other square (resp. cube) block. In the following sections, we adopt the second method in terms of BB-MAT to be used for our work throughout this paper. Similar to [9,27], we define the 2D BB-MAT of an image M as follows:

$$\mathrm{MAT}(m(i,j)) = \begin{cases} \text{the height of the largest square} & \\ \text{in the lower}-\text{right direction}, & \text{if } m(i,j)\in B_1; \\ 0, & \text{if } m(i,j)\in B_0 \end{cases} \quad (1-2)$$

An example for the chessboard distance transform and the block-based medial axis transform of a binary image M of size 8×8 is shown in Fig. 1.1. In Fig. 1.1(a), a 1-pixel is a foreground pixel, and an empty circle (0-pixel) is a background pixel. In Fig. 1.1(b), we can observe that all background pixels are the complement image of foreground pixels in Fig. 1.1(a). In Fig. 1.1(c), we can observe that all background pixels are the same as those in Fig. 1.1(a). An image of meaning “North” in Chinese is shown in Fig. 1.2(a), where the pixels with labeling numbers based on Eq. (1-2) are all foreground pixels, and the white pixels are all background pixels. (Remarks: In Fig. 1.2(a), for each pixel marked with asteroid symbol only if the MAT($m(i,j)$) is not included by any other MAT($m(i_x, j_x)$), where $i_x \le i$ and $j_x \le j$. That is, max{MAT($m(i-1, j-1)$), MAT($m(i-1, j)$), MAT($m(i, j-1)$)} ≤ MAT($m(i,j)$) [9,27].) The complement image of Fig. 1.2(a) is shown in Fig. 1.2(b), where the black pixels with labeling numbers are the chessboard distance value based on the chessboard distance transform (CDT). The “North” images of MAT and CDT based on Fig. 1.2(a) and (b) are shown in Fig. 1.2(c), and Fig. 1.2(d), respectively. Note that the pixels with black area (a skeleton) in Fig. 1.2(c) correspond to those with asteroid symbol in Fig. 1.2(a). Hence, there exist some equivalent properties between CDT and BB-MAT. Let M' be the inverse image of M, where a pixel $m(i,j)$ of M corresponds to a pixel $m'(i,j)$ of M', where $m'(i, j) = 0$ (resp. 1) if $m(i, j) = 1$ (resp. 0). One of the significant properties is CDT($m(i,j)$) for image M is equal to BB-MAT ($m'(i,j)$) for image M'. Refer to the complete works for both 2D CDT and 2D BB-MAT proposed by Lee and Horng [37]. In this paper, the relationship between 3D CDT and 3D BB-MAT is first derived and proved in Section 5.2.

1.3. 3D MAT and 3D CDT work

We list and survey some of recent works on 3D MA computations as follows: (1) Leymarie and Kimia [38] proposed the 3D medial scaffold algorithm based on their early works of the 3D shock scaffold [39], and the 3D shock hypergraph [22]. For computing the medial scaffold, it is based on a notion of propagation along the scaffold itself, starting from the initial sources of the flow and constructing the scaffold during the propagation. (2) T. K. Dey and S. Goswami [16] proposed the Tight Cocone algorithm that works on an initial mesh generated by a popular surface reconstruction algorithm and fills up all holes to output a water-tight surface. It can also compute the medial axis representation [17] that approximates the medial axis from the Voronoi diagram of a set of sample points. (3) N. Amenta and M. Bern [1] proposed the algorithm which utilizes Voronoi vertices to remove triangles from the Delaunay triangulation for computing a piecewise-linear approximation of a smooth surface from a finite set of sample points. (4) Wang and Horng [66,67] showed that the task of computing the 3D DB-MAT of a binary image of size $N \times N \times N$ can be performed in $O(1)$ time using $N^{3+\delta+\varepsilon}$ processors on the CRCW PRAM, and in $O(1)$ time using $N^{3+\varepsilon}$ processors on the array with reconfigurable optical buses (AROB), respectively, where $0 < \delta$, $\varepsilon < 1$. Later, Wang [64] proposed the 3D BB-MAT of a binary image of size $N \times N \times N$ in $O(1)$ time on an LARPBS of size max{N^3, $N^{4+\delta}$} processors, where $0 < \delta < 1$. The worst case would result in using $N^{4+\delta}$ processors. Lin and Horng et al. [41] developed a parallel algorithm that can solve the 3D BB-MAT of a binary image of size $N \times N \times N$ in $O(1)$ time using N^4 processors on the AROB.

Up to now, only the preliminary version [42] of this paper and Wang's [65] work can solve the problem of 3D CDT based on their 3D BB-MAT algorithms, respectively. In this paper, the relationship between 3D CDT and 3D BB-MAT is first derived and proved. Assume a 3D $N \times N \times N$ binary image V, with $v(i, j, k) \in \{0,1\}$, $0 \le i, j, k \le N-1$, where 1 denotes a foreground voxel, 0 denotes a background voxel, and the voxel $v(0, 0, 0)$ is located at the top-left-front corner of the 3D binary image V. Let V' be the inverse image of V; that is, a voxel $v(i, j, k)$ of V corresponds to a voxel $v'(i, j, k)$ of V', where $v'(i, j, k) = 0$ if $v(i, j, k) = 1$ and $v'(i, j, k) = 1$ if $v(i, j, k) = 0$. One of the significant properties is CDT($v(i, j, k)$) for image V is equal to BB-MAT($v'(i, j, k)$) for image V'. It can be a good motivation to find the relationship of properties connected between 3D CDT and 3D BB-MAT as stated in Section 5.2.

In a parallel algorithm, a cost is defined as the product of the time complexity and the number of processors used. The main contribution