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Defuzzification of spatial fuzzy sets by feature distance minimization

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ABSTRACT

We present a novel defuzzification method, i.e., a mapping from the set of fuzzy sets to the set of crisp sets, and we suggest its application to image processing. Spatial fuzzy sets are, e.g., useful as information preserving representations of objects in images. Defuzzification of such a spatial fuzzy set can be seen as a crisp segmentation procedure. With the aim to provide preservation of selected quantitative features of the fuzzy set, we define the defuzzification of a fuzzy set to be a crisp set which is as close as possible to the fuzzy set, where the distance measure on the set of fuzzy sets, that we propose for defuzzification, incorporates selected local and global features of the fuzzy sets. The distance measure is based on the Minkowski distance between feature representations of the sets. The distance minimization, performed in the suggested defuzzification method, provides preservation of the selected quantitative features of the fuzzy set. The method utilizes the information contained in the fuzzy representation for defining a mapping from the set of fuzzy sets to the set of crisp sets. If the fuzzy set is a representation of an unknown crisp original set, such that the selected features of the original set are preserved in the fuzzy representation, then the defuzzified set may be seen as an approximate reconstruction of the crisp original. We present four optimization algorithms, exhibiting different properties, for finding the crisp set closest to a given discrete fuzzy set. A number of examples, using both synthetic and real images, illustrate the main properties of the proposed method. An evaluation of both theoretical aspects of the method, and its results, is given.

1. Introduction

In the analysis of images it is essential to be able to distinguish between the objects of interest and the background. Traditionally, this segmentation is performed by defining crisp subsets of the image, representing different image components. However, by such a crisp segmentation it is difficult to capture uncertainty and vagueness appearing in the images. This often leads to a lack of precision and stability in the object description, where a small change in imaging conditions can cause a large change in the resulting object measures.

Fuzzy set theory and fuzzy techniques [1] have found a promising field of application in digital image analysis. Fuzzy sets fit well into situations where image components cannot easily be crisply defined, but rather in terms of their diffuse localization and extent. Fuzziness is an intrinsic quality of images and a natural outcome of many imaging techniques [2]. It appears due to several reasons: as a consequence of the imaging technique (e.g., blurring and different artifacts); as a consequence of the discretization of the data (e.g., partial area/volume coverage of image elements, leading to their partial belongingness to the image components); and as an expression of inherent vagueness of the observed objects (e.g., the image of the flame of a candle). Fuzzy concepts, incorporated in image segmentation techniques, are a useful tool for reducing the loss of data that is caused by hard decisions in the object definition. Image objects are usually defined as spatial fuzzy subsets on the integer grid, where each element of the grid is assigned a value reflecting its belongingness to the object.

A crisp representation is, however, still often needed. Reasons for that are, e.g., to facilitate easier visualization and interpretation. To visually interpret fuzzy structures in an objective way is a difficult task. Even though it contains less information, a crisp representation is usually easier to interpret and understand, especially if the spatial dimensionality of the image is higher than two. Moreover, analogues for many tools available for the analysis of binary images are still not developed for fuzzy images, which may force us to perform at least some steps in the analysis process by using a crisp representation of the image.

The process of replacing a fuzzy set by a crisp representation is referred to as *defuzzification*, see, e.g., [3,4]. This definition allows the defuzzification of a set to be any crisp subset of the reference set. In practise, different restrictions are imposed, to find a crisp representative that is intuitively suitable. Whereas most of the literature mentioning defuzzification considers defuzzification of a fuzzy set to a single (crisp) point, defuzzification of a fuzzy set to a crisp set,

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especially with an application to image processing, is not at all well explored.

Two views of defuzzification can be observed in the literature: (i) as a mapping from the set of fuzzy sets to the set of crisp sets, where some predefined criteria are respected, and (ii) as an inverse to fuzzification, where it is desirable to find a good (crisp) representation of an original continuous object. By defining the main defuzzification criterion to be the preservation (in the defuzzification) of properties which are assumed to be preserved under fuzzification, we perform defuzzification in the sense of (i), with the intention to obtain a result in the sense of (ii).

Defuzzification of a fuzzy segmented image can be seen as an alternative to crisp segmentation, where, instead of crisp segmentation of a grey-level image, fuzzy segmentation is performed, and then followed by defuzzification. It is noticed that the defuzzification of a fuzzy segmented image can provide very good crisp segmentation results [5]. In this paper, we propose a defuzzification method that generates a discrete crisp representation, of a fuzzy discrete object, utilizing local and global information extracted from the fuzzy discrete representation. It is shown in [6,7] that estimates of the perimeter and the area of a continuous object are more precise if obtained from a fuzzy segmentation of the object, instead of from a crisp segmentation. Analogous results are proved for the moments of order up to two [8]. If a crisp continuous object is represented by a fuzzy discrete representation, the precise estimates of its relevant features, together with the membership function of the fuzzy set, can be utilized to generate, by the proposed defuzzification method, a crisp digital object whose features highly resemble those of the original continuous object. It is our belief that the presented method exhibits a combination of desired properties not previously described in the literature.

The paper is organized as follows: In the next section, we introduce the main concepts used in the paper. We give a brief overview of existing defuzzification approaches and existing distance measures between spatial fuzzy sets, with the emphasis on the application in image analysis. Section 3 presents the suggested defuzzification method from a theoretical point of view. In Section 4, we discuss practical aspects of the method in the context of image processing and describe algorithms for computer implementation of the method. Section 5 contains evaluation of the method, considering both its theoretical and practical aspects. Section 6 summarizes the paper. In Appendix A, pseudo-code for two of the described search algorithms is given.

2. Background

In this section, we present existing results related to defuzzification and to distance measures between fuzzy sets, put into the context of our work. We also give a list of definitions used in the paper.

2.1. Basic notions

A *fuzzy set* S on a reference set X is a set of ordered pairs $S=\{(x,\mu_S(x))|x\in X\}$, where $\mu_S:X\to[0,1]$ is the *membership function* of S.

We denote by $\mathcal{F}(X)$ the set of fuzzy sets on a reference set X and by $\mathcal{P}(X)$ the set of crisp subsets of a set (the power set). Note that $\mathcal{P}(X)\subset\mathcal{F}(X)$.

Being interested in applications in digital image analysis, we consider digital fuzzy sets on a finite reference set $X\subset\mathbb{Z}^n$. To simplify notations we assume that $X\subset\mathbb{Z}^2$ and is defined by $X=\{(i,j)|i=1,\ldots,m,j=1,\ldots,n\}$. The cardinality of the set X is $|X|=K=m\cdot n$. A fuzzy set $S\subset\mathcal{F}(X)$ is alternatively represented by a vector of its membership values $\mathbf{s}=(s_1,s_2,\ldots,s_K)$, where $s_{i+(j-1)m}=\mu_S((i,j))$. In addition, when using digital approaches (computers) to represent, store, and analyze images, the (finite) number, $\ell+1$, of grey-levels available is a natural limitation to the number of membership values that can be assigned to a digital point.

An *α-cut* of a fuzzy set S, for $\alpha\in(0,1]$, is the set $S_\alpha=\{x\in X|\mu_S(x)\geq\alpha\}$. The *support* of a fuzzy set S is the set $Supp(S)=x\in X|\mu_S(x)>0$. The *core* of a fuzzy set S is the set $Core(S)=\{x\in X|\mu_S(x)\geq\mu_S(y)\text{for all } y\in X\}$.

The *fuzzification principle*, based on

$$f(S)=\int_0^1\hat{f}(S_\alpha)d\alpha, \tag{1}$$

can be used to generalize properties $\hat{f}$, defined for crisp sets (here, α-cuts), to fuzzy sets. For generalizing a property $\hat{f}$ to fuzzy sets defined using a finite number of equidistant membership levels, the equation

$$f(S)=\frac{1}{\ell}\sum_{\alpha=1}^{\ell}\hat{f}(S_\alpha), \tag{2}$$

can be used instead. In other words, the value of the (crisp) feature f of a fuzzy set S is defined as an average of the values of the corresponding feature $\hat{f}$ of the α-cuts of the fuzzy set S.

In this paper, we use Eq. (2) to define *perimeter* $P(S)$, of a fuzzy set S. More detailed definition, properties, and efficient implementation are given in [6]. Eq. (2) is also used to define *geometric moments of zero and first order* of a discrete spatial fuzzy set S, denoted by $m_{p,q}(S)$. More detailed definitions and properties are described in [8].

The area of a set S, $A(S)$, is equal to the zero-order moment of the set, $m_{0,0}(S)$. The centroid of a set S is $(C_x(S),C_y(S))=\left(\frac{m_{1,0}(S)}{m_{0,0}(S)},\frac{m_{0,1}(S)}{m_{0,0}(S)}\right)$.

For $x,y\in\mathbb{R}^n$, the Minkowski distance of order p is

$$d_p(x,y)=\left(\sum_{i=1}^{n}|x_i-y_i|^p\right)^{1/p},\quad p\geq1, \tag{3}$$

$$d_\infty(x,y)=\max_{i=1,\ldots,n}(|x_i-y_i|). \tag{4}$$

On a finite-dimensional space, the Minkowski distance is a metric for $p\geq1$.

2.2. Related work on defuzzification

Defuzzification methods are often designed and evaluated with respect to some criteria that they should fulfil. One such collection of criteria for defuzzification of a fuzzy set to a (crisp) point is formulated in [3] and applied in the evaluation of several widely used defuzzification techniques, whereas a set of criteria, which should be fulfilled in the process of replacing a fuzzy set by a crisp set, is formulated in, e.g., [4]. In both cases, criteria are related to: (i) if defuzzification considers only points with highest memberships or takes into account additional information from other fuzzy set elements; (ii) if defuzzification commutes with certain transformations, both in spatial and membership domain; (iii) if certain monotonicity properties are preserved under defuzzification.

A purely theoretical approach, where defuzzification is seen as an inverse of fuzzification, is presented in [9]. An optimal defuzzification is defined to restore the original object, after a given fuzzification has been applied to it. It appears to be very difficult to derive the inverse of a fuzzification, even if the membership function leading to it is analytically defined. Solutions are given only for a limited number of simple membership functions [9].

When imaging, in combination with a fuzzy segmentation, provides a digital fuzzy representation of a crisp continuous object, the obtained membership values are, to a high extent, dependent on the imaging device and the imaging conditions. The fuzzification rules are, therefore, usually either not known, or very difficult to reconstruct and use in practise. Consequently, a method to recover a continuous crisp object from its fuzzy representation can rarely be