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Shock filter coupled to curvature diffusion for image denoising and sharpening

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ABSTRACT

The frequent problem in low-level vision arises from the goal to eliminate noise and uninteresting details from an image, without blurring semantically important structures such as edges. Partial differential equations (PDEs) have recently dominated image processing research, as a very good tool for noise elimination, image enhancement and edge detection. In this paper, we present a biased PDE filter based on a coupling between shock filter and curvature diffusion. This model removes noise and sharpens edges efficiently. It preserves well the location of the shocks by synchronising both effects of smoothing and deblurring. Empirical results on different kinds of images confirm these advantages.

1. Introduction

Image restoration is an important step in low-level computer vision especially when the input image is blurred, noisy or blurred and noisy. An ideal restoration algorithm is expected to simultaneously remove noise and enhance edges in an image [1,2]. However, it is well-known that both noise and edges are high frequency signals. Therefore, it is very difficult to remove one while retaining the other. Hence, many researches using a selective smoothing operation have been developed in order to solve this problem, and some satisfactory results have been reported [3,4]. Among the existing restoration algorithms, the approach that bases a selective smoothing procedure on partial differential equations (PDEs) has attracted a lot of attentions in recent years [5–8]. In this context, a large number of techniques based on PDEs has been proposed. The elliptic methods are frequently related to variational problems via Euler equations [9–11] and shock filter [12–14]. While parabolic field is governed by diffusion process represented by mean curvature [15,16] and anisotropic diffusion [3,17–19].

Hence, shock filter belongs to the class of morphological image enhancement methods, that offers a number of advantages. It creates strong discontinuities at image edges, and within a region the filtered image becomes flat. A sharp shock between two influence zones is created [12]. This filter is based on the idea to apply locally either a dilation or an erosion process, depending on whether the pixel belongs to the influence zone of a maximum or a minimum. In the other hand, diffusion method is an efficient nonlinear filtering technique for performing contrast enhancement and noise reduction [3,20]. Central to the technique is a diffusivity or edge stopping function, that controls the degree of image smoothing. It depends on the choice which edges have to be enhanced, and which have to be cancelled.

By exploiting the image enhancement propriety of the first method and the selective smoothing operation of the second, we present a contribution based on a biased ideal solution estimation of blurry and noisy images, where shock filter is coupled to curvature diffusion.

2. Overview

The 2D formulation of the classical shock filter with reflecting boundary conditions is commonly given by [12]:

$$u_t = -\mathrm{sign}(u_{\eta\eta})u_\eta \tag{1}$$

where $u(t)$ is the image solution at time t and $u(0) = u_0$ the input image. η is the direction of the gradient ∇u with:

$$\begin{cases} u_{\eta\eta} = \frac{u_{xx}u_x^2 + 2u_{xy}u_x u_y + u_{yy}u_y^2}{u_x^2 + u_y^2} \\ u_\eta = |\nabla u| = \sqrt{u_x^2 + u_y^2} \end{cases} \tag{2}$$

The main idea assumes that some pixel is in the influence zone of a maximum, where its second derivative $u_{\eta\eta}$ is negative. Then (1) becomes:

$$u_t = |\nabla u| \tag{3}$$

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Evolution under this PDE is known to produce at time t a dilation process with a disk-shaped structuring element of radius t. At the influence zone of a minimum with $u_{\eta\eta} > 0$, Eq. (1) can be reduced to an erosion equation with a disk-shaped structuring element:

$$u_t = -|\nabla u| \tag{4}$$

These considerations show that for increasing time, (1) increases the radius of the structuring element until it reaches a zero-crossing of $u_{\eta\eta}$, where the influence zones of a maximum and a minimum meet. Therefore, the zero-crossings of the derivative $u_{\eta\eta}$ serve as an edge detector, where a shock is produced that separates adjacent segments [21].

However, as mentioned in the original paper, any noise added to the signal may create an infinite number of infection points disrupting the process completely. Hence, Alvarez and Mazorra [22] replaced the edge detector $u_{\eta\eta}$ by its convolution with the Gaussian function G_σ, where σ is the standard deviation of the Gaussian. The filter becomes more robust against noise, but not sufficiently efficient. Taking into account this modification, the shock filter becomes:

$$u_t = -\text{sign}(G_\sigma * u_{\eta\eta})u_\eta \tag{5}$$

Thus, a simplest isotropic diffusion filter eliminates noise, but also blurs and dislocates important features in the image such as the edges. Therefore, when moving from the finer to coarser scale, the image gets more and more simplified and the edges are dislocated [23]. Both effects of blur and dislocation of edges can be avoided by inhibiting diffusion along edge direction, while preserving an isotropic behaviour within stationary level grey region. This anisotropy idea has been extensively used as an efficient algorithm for simultaneously performing contrast enhancement and noise reduction. The Perona–Malik model has stimulated a great deal of interest in image selective smoothing field. The basic idea behind this algorithm is to evolve an input image u_0 under an edge-controlled diffusion operator [3]. Regularised version with reflecting boundary conditions is given by [20]:

$$u_t = \text{div}(g(|\nabla u_\sigma|)\nabla u) \tag{6}$$

$u(0) = u_0$ and ∇u_σ is the gradient of a Gaussian-smoothed version of u.

$g(s)$ is monotonically decreasing function in gradient magnitude and usually contains a free parameter, k_d, that determines the contrast of edges that will have significant effects on the smoothing. It is of the form:

$$g(s) = \frac{1}{1 + \frac{s^2}{k_d^2}} \tag{7}$$

Anisotropic behaviour results in distinguishing between intra- and inter-region smoothing. It returns the zones with weak gradient magnitude homogeneous eliminating noise and reduces the diffusion along edges in order to preserve them. The parameter k_d is small for large edge-strength and vice versa. The histogram of the gradient values may give some clue about edges distribution [5].

More complex and efficient diffusion filter at edge enhancing and noise removing has been proposed by Whitaker and Xinwei [15]. It is based on the curvature diffusion operator of [24] and the geodesic active contours method of [25] and given by:

$$u_t = |\nabla u|\text{div}\left(g(|\nabla u_\sigma|)\frac{\nabla u}{|\nabla u|}\right) \tag{8}$$

The authors used the following reasoning: edge enhancing properties of anisotropic diffusion can be examined by decomposing the diffusion term into local coordinates η and ξ, that are aligned with the gradient and level-set directions of the image. Thus, the diffusion from (6) becomes:

$$u_t = (g' u_\eta + g)u_{\eta\eta} + g u_{\xi\xi} \tag{9}$$

where

$$u_{\xi\xi} = \frac{u_{xx}u_y^2 - 2u_{xy}u_x u_y + u_{yy}u_x^2}{u_x^2 + u_y^2} \tag{10}$$

A similar analysis of (8) gives:

$$u_t = (g' u_\eta)u_{\eta\eta} + g u_{\xi\xi} \tag{11}$$

Hence, the diffusion in the gradient direction is always negative. Because the function g is chosen to be monotonically decreasing. The net smoothing behaviour is defined by the relationship between $u_{\eta\eta}$ and $u_{\xi\xi}$ which is small for straight edges. Thus, the resulting equation is more dependent on the size of the objects that form that edges rather than their contrast [15]. Referring to these advantages, we will be interested in our proposition by the last diffusion model.

3. Related works

Alvarez and Mazorra have defined a new class of filter for noise elimination and edge enhancement by coupling shock filter to the diffusion operator. Thus, smoother parts are denoised, whereas edges are sharpened [22]. The principle is to add some kind of anisotropic diffusion term with an adaptive weight between the shock effect and the diffusion process. This filter is given by the following PDE:

$$u_t = Cu_{\xi\xi} - \text{sign}(G_\sigma * u_{\eta\eta})u_\eta \tag{12}$$

where C is a positive reel constant.

The parabolic–hyperbolic PDE Eq. (12) diffuses the initial image in the direction perpendicular to the gradient eliminating noise, and develops a shock in the direction parallel to the edge sharpening and enhancing edges. The term $u_{\xi\xi}$ makes the image smooth on both sides of the edge with a minimal smoothing of the edge itself, where the second term $-\text{sign}(G_\sigma * u_{\eta\eta})\ u_\eta$ produces edge enhancement in the position of zero-crossing of $(G_\sigma * u_{\eta\eta})$. The constant C is used as a balance between anisotropic diffusion behaviour and shock effect [22]. This balance has been more investigated by Kornprobst in the following scheme [26]:

$$u_t = \alpha_f(u - u_0) + \alpha_r(hu_{\eta\eta} + u_{\xi\xi}) - \alpha_e(1-h)\text{sign}(G_\sigma * u_{\eta\eta})u_\eta \tag{13}$$

with $h = 1$ if $|\nabla u_\sigma| < k$ and 0 otherwise, α_f, α_r and α_e are some constants. The first term on the right is a fidelity term to carry out a stabilization effect. Thus, by choosing adequate values of k, α_r and α_e the second term behaves like a denoising operator, where the last one assures the enhancement task. In the case of $h = 1$, this scheme can be viewed as a coupling between shock filter and linear diffusion with Laplacian operator, which diffuses the image in the gradient and level set directions. In the other hand, it will be similar to Alvarez–Mazorra model.

However, a new form of coupling shock filter to diffusion process has been proposed [7]. Motivated by quantum mechanics and Schrödinger equation, Gilboa developed a generalised complex shock filter for image enhancement and a ramp preserving deblurring and denoising. It is based on adding a complex diffusion term to the shock equation. This new term is used to smooth out noise and indicate inflection points simultaneously. The imaginary value of the solution, which is an approximated smoothed second derivative, is used to control the process. For a small time, the filter operates as a diffusion process smoothing noise and with large time shocks are created. This filter is given by:

$$u_t = -\frac{2}{\pi}\arctan\left(a\ Im\left(\frac{u}{\theta}\right)\right)u_\eta + \lambda u_{\eta\eta} + \tilde{\lambda}u_{\xi\xi} \tag{14}$$