Image and Vision Computing 26 (2008) 1499–1506

Contents lists available at ScienceDirect

Image and Vision Computing

journal homepage: www.elsevier.com/locate/imavis

Contour simplification using a multi-scale local phase analysis

F.J. Madrid-Cuevas *, A. Carmona-Poyato, R. Medina-Carnicer, R. Muñoz-Salinas

Dpto. de Informática y Análisis Numérico, Universidad de Córdoba, Campus de Rabanales, s/n, 14071 Córdoba, Spain

ARTICLE INFO

Article history:
Received 12 September 2007
Received in revised form 21 January 2008
Accepted 16 April 2008

Keywords:
Contour simplification
Local phase
Multi-scale contour analysis

ABSTRACT

This paper proposes a new method for simplifying contours based on the multi-scale analysis of the local phase. The main advantages of the proposed method are: (i) it does not use any curvature measure approximation to stand out the characteristic points. (ii) The symmetry/asymmetry points can be considered as dominant points of the contour and (iii) it provides a robust approach to suppress the contour noise. The method has been compared with a representative number of other methods using an objective measure of the quality of the generated approximation. The experimental results have shown that the proposed method is superior to those reviewed in our study.

1. Introduction

Contour Matching is a fundamental tool in Shape Analysis. Many contour representations have been proposed in the relevant literature. Zhang et al. [1] reviewed and classified shape representations and description techniques including contour representations based on a coordinates system [2,3] (Cartesian coordinates, polar coordinates, polar coordinates in relation to the centroid and tangential representations); chain code representation [4–6]; signature representation [7,3]; arc – height function (AHF) representation [8,2]; polygonal representations [9,4,10]; splines and B-splines representations [11–13] and hierarchical representations [14,15].

A very interesting subject in contour matching is contour simplification that preserves the original characteristics of shape features. This simplification process can be described as the partition of a contour into meaningful parts [16]. Contour partition can be divided into two generic phases. The first phase determines the segmentation points along the contour, while the second phase represents each segment in terms of instances of a predefined geometric primitive. Since the simplest and most commonly adopted primitives are straight segment lines, the output of such a process is a polygonal approximation of the original contour. The segmentation points of the original contour that define the polygonal approximation are commonly called Dominant Points.

Dominant point detection is an important research area in contour approximation methods. Many algorithms are used to detect dominant points. These methods can be classified into three categories [17]:

- Methods which search for dominant points using some significant measure other than curvature from the original contour scale [18–20,12,5,21–25] or from a multi-scale contour representation [26–29]. The multi-scale methods take into account [26] the well accepted fact that image features occur in a broad rang of scales.
- Methods which evaluate the curvature by transforming the contour to the Gaussian scale space [30–32].
- Methods which search for dominant points by directly estimating the curvature in the original picture space [33–35].

Based on the review of the proposed methods, the following conclusions were drawn:

- Generally only corner points (extreme curvature) are considered dominant points.
- Few authors analyse the problem of noise. Of the papers we reviewed, none evaluated performance with noisy contours.

In this paper a new contour simplification method using a polygonal approximation is proposed. The approximation vertices will be the dominant point of the contour. In the revised literature, in general, only high curvature points are considered as dominant points. However [36], high curvature points may not provide good approximations for smooth curves.

From the local phase analysis, also we can detect local symmetry/asymmetry points. These points with the local phase congruence points will form a candidates set to be the dominant points. A method is proposed to select the sub-set of dominant points that provides a very good trade-off between minimum distortion and maximum compression rate by minimising the E2 objective function used by Carmona et al. [37].

* Corresponding author. Tel.: +34 957 211035; fax: +34 957 218630.
E-mail address: ma1macuf@uco.es (F.J. Madrid-Cuevas).

0262-8856/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.imavis.2008.04.009

In Section 2, the multi-scale analysis of the local phase is described. The proposed method for dominant points detection and the procedure to obtain the sub-set of dominant points that defines the polygonal approximation are shown in Section 3. In Section 4 the results of three experiments to evaluate method performance with and without noise and a comparative study with a representative number of proposed methods are shown. Lastly, the main conclusions are summarised in Section 5.

2. Local phase feature

The information provided by the local phase of a signal serves as the basis of our method for two reasons:

- The phase is a dimensionless quantity that allows scale invariant characteristics to be developed.
- The phase of a signal has been shown to be crucial in shape perception [38].

Fig. 1 shows a triangular wave together with its expansion in its first Fourier terms. Notice how the Fourier components of the wave are all in phase at the corner point of the wave. The points at which the local phase of the signal components coincides are called High Phase Congruence Points.

Given a point $C(t)$ of a signal, the phase congruence at this point can be obtained from the Fourier series expansion of the signal as follows [39]:

$$\mathrm{PC}(t) = \max_{\bar{\phi}(t)\in[0,2\pi]} \frac{\sum_n A_n \cos(\phi_n(t) - \bar{\phi}(t))}{\sum_n A_n}, \tag{1}$$

where A_n and $\phi_n(t)$ represent the amplitude and the local phase of the n-th Fourier term, respectively. The value $\bar{\phi}(t)$ that maximises $\mathrm{PC}(t)$ is the amplitude weighted mean local phase of all the Fourier terms at the point being considered.

An initial approach to calculate the phase congruence could be to use the Fourier transform of the signal. However, this approach has two main drawbacks:

- The importance of a signal feature is compared to the complete signal (great scale) without taking into account the signal feature's importance regarding its most immediate environment (small scale).
- The number of congruent terms at a point is not taken into account. The larger the number of congruent terms at a signal point, the more outstanding the signal feature will be.

One way to address these problems is to use a multi-scale analysis of the local phase. Kovesi [40] proposes using a bank of even/odd filters to obtain the local energy of a signal with spatially localised frequency.

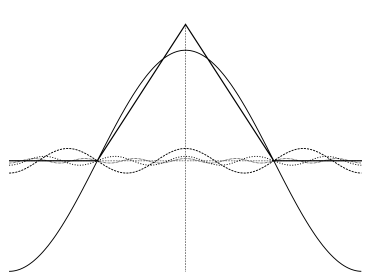

Fig. 1. First few terms of a triangular waveform. Notice how the Fourier components are all in phase at the corner points.

2.1. Calculation of the phase congruence using wavelets

The wavelet analysis of a signal allows spatially localised frequency information to be obtained in a very precise way. The wavelet analysis uses a filter bank that is created from re-scalings of a wave shape. Each scaling is designed to analyse a given range of signal frequencies. In order to preserve phase information, lineal phase filters should be used, that is to say, quadrature filters.

Let $M : \{(M_n^e, M_n^o)\}$, $n = \{0, 1, \ldots, N\}$ the bank of quadrature filters where n represents the scale parameter and N is the number of analysed scales. Using this filter bank the following terms can be computed:

$$e_n(t) = C(t)^* M_n^e, o_n(t) = C(t)^* M_n^o, \tag{2}$$

$$\sum\nolimits_N A_n(t) = \sum_N \sqrt{e_n(t)^2 + o_n(t)^2}, \tag{3}$$

where * represents the digital convolution operation.

As stated above, a signal feature will be more important if it is present in a larger number of analysed scales. To make these points stand out, Kovesi proposes weighting using the following sigmoid function:

$$W(t) = \frac{1}{1 + e^{10(0.4 - s(t))}}, \tag{4}$$

where $s(t)$ is a measure of the range of congruent frequencies at a point t of the signal and is defined as:

$$s(t) = \frac{1}{N}\left(\frac{\sum_N A_n(t)}{A_{\max}(t) + \epsilon}\right), \tag{5}$$

where N is the number of analysed scales, $A_{\max}$ is the maximum filter bank response obtained and ϵ is a small quantity used to avoid division by zero.

Other drawback to calculating the phase consistency by means of (1) is that it is proportional to the cosine of the phase angle deviation $\phi_n(t)$ from the overall scales mean phase angle $\bar{\phi}(t)$. The cosine function is not very sensitive to small variations around zero, for example $\cos(25^\circ) \approx 0.9$. This implies that a poor localisation of the signal features can be provided. To improve localisation, Kovesi proposes using a measure of the deviation from the phase angle which is more sensitive to small variations:

$$\Delta\Phi_n(t) = \cos(\phi_n(t) - \bar{\phi}(t)) - |\sin(\phi_n(t) - \bar{\phi}(t))|, \tag{6}$$

providing a second approach to calculate the phase congruence:

$$\mathrm{PC}_2(t) = \frac{W(t)\sum_N A_n(t)\Delta\Phi_n(t)}{\sum_N A_n(t) + \epsilon}. \tag{7}$$

Eq. (7) can be calculated from the quadrature filter responses. For each scale n, the filter response can be considered as a vector $(e_n(t), o_n(t))$ whose magnitude is $A_n(t)$. The unitary vector that provides the direction of the overall mean phase angle is given by:

$$(\bar{\phi}_e(t), \bar{\phi}_o(t)) = \frac{1}{\sqrt{\left(\sum_N e_n(t)\right)^2 + \left(\sum_N o_n(t)\right)^2}}\left(\sum_N e_n(t), \sum_N o_n(t)\right). \tag{8}$$

2.2. Symmetry and asymmetry features

It has been indicated in the literature [36] high curvature points (corner points) may not provide good approximations, mainly for smooth curve segments. We propose useful information can also be obtained from the points that coincide with local symmetry or asymmetry axes of the contour. These points allow to make better approximations for smooth curve segments.

For example, Fig. 2 shows a semicircle curve. This curve has not curvature maxima and the local phase congruence for all the curve