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Dual fractals

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Abstract

This paper presents some definitions and propositions concerning to dual fractals. Among them, *dual-similarity* plays a key-role not only in generating dual fractals but also in handling inter-pattern relations. Dual-similarity is basically defined as a pair of the similarity relations between two patterns, from which two mirror operators have been derived. This paper shows that each mirror operator is nothing but a contraction mapping associated with a unique attractor. Next, the mirror operator has been extended to *ring mapping* defined as a cyclic sequence of contraction mappings for a sequence of patterns. Basic experiments have been carried out, correlating with some application schemes, to verify the obtained theoretical outcomes in the sense of approximation to the truth.

Keywords: Dual fractals; Dual-similarity; Hutchinson operator; Image coding; Template matching; Secret sharing; Feature extraction

1. Introduction

Fractal geometry [1] has been attractive for a long time for people engaged in computer graphics, pattern recognition, image coding and other related subjects. In relation to such subjects, many fractal applications have already been presented [2–11]. Among them, the fractal image compression technique [3,5] has given a strong impact on studies on fractals not only in terms of image coding but also in terms of practical understanding of fractals. Especially *MRCM* (*Multiple Reduction Copy Machine*) or *IFS* (*Iterated Function System*) has given a key to open doors towards breaking fresh ground in fractal geometry.

As is well known, every fractal pattern is characterized by a set of similarity relations between the entire pattern and its parts as seen in the well-known Koch curve (see Fig. 1). Mathematically this is termed *self-similarity*. Either MRCM or IFS can be regarded as a procedural representation of self-similarity, by which a fractal pattern is to be defined as the limit pattern. It is a very suggestive example that the Sierpinski gasket, one of the well-known fractal patterns, can be explained as the limit pattern resulting from running a simple MRCM infinitely many times [3]. An MRCM, in short, corresponds to a set of self-similarity relations to determine a fractal pattern. Hutchinson represented an MRCM by a set of contraction mappings, which is termed *Hutchinson operator* [12]. This operator plays an important role in extending self-similarity to different phases.

Self-similarity is nothing but a set of inner relations defined within *one* fractal pattern, so that it has been isolated from mutual relations between patterns; i.e. inter-pattern relations. To discuss such relations in terms of fractal geometry, we have to build a bridge across a gap between self-similarity and inter-pattern relations. We introduce the *dual-similarity* that is the very thing compared to the bridge.

In Section 2, we first organize a collection of well-known definitions and theorems about fractals towards introduction of dual-similarity. Meanwhile, a pair of two intimate fractal patterns, termed *dual fractals*, derived from dual-similarity has been introduced [13]. In Section 3, we have described the dual-similarity by a pair of Hutchinson operators: Since dual fractals can also be explained by MRCM associated with self-similarity, we discuss dual fractals also in connection with self-similarity. Here emphasis has been

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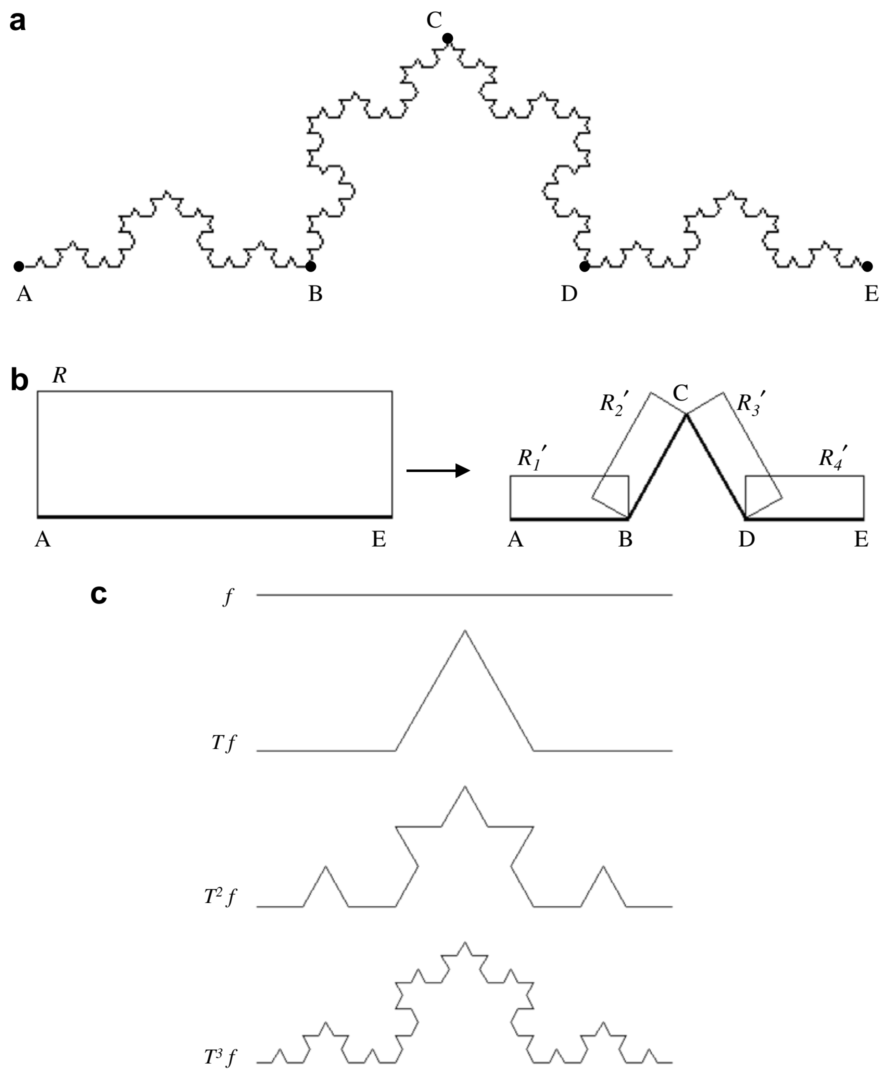

Fig. 1. Koch curve and contraction mappings: Koch curve is partitioned into the four parts each of which is a 1/3 contraction of the entire pattern. This structure, termed self-similarity, can be interpreted by a Hutchinson operator [5]. Koch curve has five definite points A, B, C, D and E shown in (a). Suppose a rectangular region R surrounding the entire pattern in (b), including a line segment AE. Here we can reduce R into four 1/3 contractions R_1', R_2', R_3' and R_4' and place them along four line segments AB, BC, CD and DE, respectively. Let T_i $(i = 1,\ldots, 4)$ be such a contraction mapping that reduces a pattern drawn in R into a 1/3 contraction and pastes it on each of $\{R_i'\}$. Then Koch curve can be given by attractor q of a Hutchinson operator $T = T_1 \cup T_2 \cup T_3 \cup T_4$. From Theorem 1, for a given pattern f, the sequence $\{T^n f | n = 1,2,\ldots\}$ converges to attractor q. When f is line segment AE in (b), an initial part of the sequence; f, Tf, $T^2 f$ and $T^3 f$ is shown as (c).

placed on mathematical discussion on the mutual relation between dual-similarity, dual fractals and self-similarity. Then we describe the ring mapping for a cyclic sequence of patterns, which has been introduced as extension of dual-similarity. Finally, in Section 4, we present basic experiments to verify our theoretical outcomes obtained in the first half of this paper.

2. Contraction mapping and fractals

The concept of fractals has brought together under one umbrella a broad range of preexisting concepts from pure mathematics to the most empirical aspects of engineering. It is not clear that a single mathematical definition can encompass all these applications [3], but we make too much of contraction mappings for representation of fractal generating procedures. Here we discuss the self-similarity in the light of mathematical mapping (or transformation) in the metric space. The first step towards our discussion is to put some definitions.

Definition 1. Let I be a set of real numbers $\{x | 0 \leqslant x \leqslant 1\}$. Let every pattern depicted on the continuous unit square $I^2 = \{(x_1, x_2) | x_1, x_2 \in I\}$ be defined by a real-valued function $0 \leqslant f(x_1, x_2) \leqslant 1$ that means gray-level on a point (x_1, x_2). Let $\boldsymbol{x} = (x_1, x_2)$, then a pattern is written $f(\boldsymbol{x})$ and, sometimes, denoted simply as f.

When pattern $f(\boldsymbol{x})$ is a binary pattern of which value is either 0 or 1, f is equivalent to a set of points S_f in I^2. Symbolically we can write