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Nonrigid motion recovery for 3D surfaces

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Abstract

We present a spline-based nonrigid motion and point correspondence recovery method for 3D surfaces. This method is based on differential geometry. Shape information is used to recover the point correspondences. In contrast to the majority of shape-based methods, which assume that shape (unit normal, curvature) changes are minimum after motion, our method focuses on the nonrigid relationship between before-motion and after-motion shapes. This nonrigid shape relationship is described by modeling the underlying nonrigid motion; we model it as a spline transformation, which has global control over the entire motion field along with the local deformation integrated within. This provides our method certain advantages over some pure differential geometric methods, which also utilize the nonrigid shape relationship but only work on local areas without a global control. For example, motion regularity is hard to implement in these pure differential geometric methods but is not a problem when the motion field is controlled by a spline transformation. The orthogonal parameterization requirement of the nonrigid shape relationship has to be approximated in these previous methods but is easy to meet in our method. Furthermore, the small deformation constraint introduced by the previous works is relaxed in our method.

Experiments on both synthetic and real motions have been conducted. The quantitative and qualitative evaluations of our method are presented. The application of our method to the human tongue motion analysis is also presented in this paper.

Keywords: Nonrigid motion; Correspondence; Spline; Shape-based methods

1. Introduction

Nonrigid motion estimation is an important research area in computer visions. It has many applications such as in medical image registration [4,12,28,30], face modeling [29,33] and remote sensing [34,35]. Comprehensive reviews of early nonrigid motion analysis methods can be found in Refs. [3,18]. Among the numerous motion and correspondence estimation approaches, shape-based methods have been extensively studied by several researchers [4,12,15,24,28]. Advantage of shape-based methods compared to other approaches such as *physically based methods* [11,25,26] is that motion is estimated solely from the visual data.

Although shape information provides basis for the motion and correspondence estimation problem, different constraints still need to be applied to avoid the motion and correspondence ambiguity due to the surface complexity and the nonrigid property of motion. These constraints are usually introduced by comparing some shape properties such as curvatures and unit normals, of the before-motion and after-motion surfaces. Generally, there are two approaches to compare shape properties: the direct shape-based, and the 'nonrigid' shape-based approach.

Before discussing these two approaches, we first introduce some notations used in this paper. $\mathbf{X}=(x, y, z)$ denotes the before-motion surface where (x, y, z) is a point on the surface $\mathbf{X}$, $\mathbf{X}'=(x', y', z')$ is the after-motion surface where (x', y', z') is a point on the surface $\mathbf{X}'$. $\mathbf{n}$[1] and $\mathbf{n}'$ denote the unit normals of the before-motion and after-motion surfaces, respectively. κ and κ' are the curvatures of these two surfaces, respectively. $\mathbf{S}$ denotes the displacement between these two surfaces, i.e. $\mathbf{S}=\mathbf{X}'-\mathbf{X}$.

1.1. The direct shape-based method

In general, the direct shape-based method assumes among all possible displacements, the one that minimizes the

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[1] At a given point, unit normal $\mathbf{n}$ has different values in the world and local coordinate systems. We do not explicitly distinguish these two values with different notations. The meaning of $\mathbf{n}$ depends on the context; same for the $\mathbf{n}'$.

following objective function (or part of it) as the correct estimate for point correspondences

$$\alpha_1 \sum_X \{\kappa(\mathbf{X}) - \kappa'(\mathbf{X}+\mathbf{S})\}^2 + \alpha_2 \sum_X ||\mathbf{n}(\mathbf{X}) - \mathbf{n}'(\mathbf{X}+\mathbf{S})||^2 \quad (1)$$

where α_1 and α_2 are the weighting parameters. Note that in the above formula, in order to compare normals, one has to either compute $\mathbf{n}$ and $\mathbf{n}'$ in the world coordinate system after the before-motion and after-motion surfaces have been globally aligned as in Ref. [24], or compute $\mathbf{n}$ and $\mathbf{n}'$ in a local principle coordinate system as in Ref. [13].

The above objective function is based on the invariance of the shape properties (unit normal and curvature). In Ref. [30], this shape invariance is combined with geodesic distance to determine point correspondences between surfaces. In Ref. [12] the curvature invariance is used to measure the motion of deformable objects. The shape invariance properties are also used in Ref. [13], which is based on the popular iterative closest point (ICP) algorithm [7,32] registration method but has been extended to the nonrigid situation. Some other examples of the direct shape-based method are Refs. [24,28].

The assumption of the direct shape-based method is that the shape properties do not change (much) after motion. But curvature and unit normal are only rigid invariants; in nonrigid motion situation, shape properties vary, making the rigid motion assumption invalid.

1.2. The nonrigid shape-based method

Different from the direct shape-based method, the nonrigid shape-based method [15–17,19] starts from shape relationships between the before-motion and after-motion surfaces. The relationships are not based on simple invariance, but they are based on the underlying nonrigid motion and geometric properties of surfaces (we call this kind of shape relationship as nonrigid shape relationship). Among given correspondence hypotheses on the after-motion surface, the nonrigid shape-based method tries to find the corresponding point for each point on the before-motion surface by selecting the hypothesis which best fits a pre-defined nonrigid motion model. Fig. 1 shows all correspondence hypotheses of a point, P on a surface S, which has undergone nonrigid motion to map on S'. Point P can correspond to any point within some region, R (the gray area in Fig. 1). R is the region checked for point correspondences. It is defined as a small window around the position of the point before the motion by the small motion assumption. In the estimation of motion parameters, these previous nonrigid shape-based methods also consider the neighborhood points for the error computation. That is local patches at each point under consideration. Thus, the mapping of a set of neighboring points, P_i onto another set P_i' is assumed to satisfy the nonrigid relationship between two points P and some point P'. However, the nonrigid shape relationship is valid only between point P and P' since this relationship is valid in the condition of orthogonal parameterization (see Section 2 for the definition of orthogonal parameterization) and

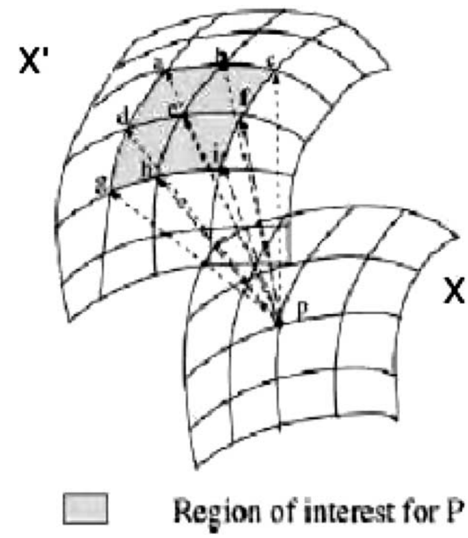

Fig. 1. The previous nonrigid shape-based method.

the parameter orthogonality can only be guaranteed at point P by constructing a local coordinate around P. Parameter orthogonality of neighboring points of P is not guaranteed in the constructed local coordinate around P. Most of the previous nonrigid shape-based methods [15–17] omit this problem and assume the orthogonal parameterizations of these neighboring points. The nonrigid shape relationship between P and P' is directly applied to the neighboring points of P. Though in Ref. [19] a curvilinear orthogonalization method is presented, the recovered motion in this paper is limited to local affine motion.

The shape relationship used in the nonrigid shape-based method is described in a local coordinate system. Thus, the motion models in Refs. [15–17,19] are all defined in different local coordinate systems for different points on the before-motion surface. These definitions introduced two problems. First, the motion defined in the local coordinate system has no explicit physical meaning. Second, the motion consistency over the entire motion field as a whole cannot be guaranteed with local motions defined in different local coordinate systems.

1.3. Our approach

Our approach is based on previous nonrigid shape-based methods [15–17,19]. We still utilize the nonrigid shape relationship defined in the *local* coordinate system around each point of interest. But instead of using the local motion model which means different motion models are defined for different points of interest on the before-motion surface, we model the nonrigid motion of all points of interest on the before-motion surface with a *single* GRBF (Gaussian Radial Basis Function) transformation (which is a spline transformation) that is defined in the world coordinate system. The nonrigid motion recovery problem is solved by estimating the parameters of a single GRBF [21]. Unlike the previous nonrigid shape-based methods [15–17,19] in which neighboring points around each point of interest are required to estimate