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Genetic Fourier descriptor for the detection of rotational symmetry

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Abstract

In this paper, a genetic Fourier descriptors is proposed to detect rotational symmetry. Rotational symmetry is one of the important features for image decoding and object recognition in computer vision systems. In the genetic Fourier algorithm, the Fourier descriptors are chromosomes and fitting function of the GA. The genetic Fourier method has the following advantages. (1) It can handle partially occurred contour and opened contour, (2) it can handle complex point pattern and (3) it can obtain multiple perceptions. Experimental results show that it can handle complex symmetry figures, these symmetry figures may be formed by separated curves, points or partially occurred or partially missed (opened contour).

Keywords: Genetic algorithm; Fourier descriptors; Rotational symmetry detection

1. Introduction

In this paper, a genetic Fourier descriptors is proposed to detect rotational symmetry. Rotational symmetry is one of the important features for image decoding and object recognition in computer vision systems. It plays an important role in pattern recognition, especially when extracting a planar symmetric figure from a single image without the need of models. In the recent years, many methods have been proposed for the detection of rotational symmetry. However, without using a predefined template (model), most of the methods are difficult to handle complex symmetry figure, e.g. Figs. 8a and 9a. Some methods, like Hough transform [1] can handle these situation but they only extract the symmetric points and the problem of finding the connected contour (connectivity problem) and equation (parameterization problem) of the figure still left open.

Fourier descriptors, FDs [2–7,13] are one of the powerful tools for object recognition and symmetry detection. However, it cannot handle partially occurred symmetry figure, symmetric point pattern and opened contour situation. FD's are useful in describing and recognizing the shapes of 2D closed contours. The basic idea is, a closed curve may be represented by a periodic function of a continuous parameter t, or alternatively, by a set of Fourier coefficients of this function. In general, there exists infinitude ways to parameterize a 2D closed curve to form the FD's. The reason for the infinitude is that t is an implicit (hidden) parameter. Therefore, there exists an infinite number of ways to distribute its values along the curve. The most common (simplest) way is based on the arc length $l(t=2\pi l/L$ where L is the perimeter of the contour) parameterization, Fig. 1a.

There are several ways to represent a curve using Fourier descriptors with arc length parameterization. In Zhan 1972 [2], a curve is represented as a function of the arc length l by the accumulated change in direction (tangent angle $\theta(l)$) of the curve. In Granlund 1972 [3], the closed contour is represented by a function of the arc length as $[X(l),Y(l)]$ in the complex plane, and the complex function $U(l)=X(l)+iY(l)$ is then expanded in a Fourier series. A similar approach, the elliptic Fourier features [4], expands $X(l)$ and $Y(l)$ separately and put them in matrix form, Eq. (1) and Fig. 1b.

On the other hand, genetic algorithms (GAs) [9–12] are powerful tools in the areas of computer vision. It employs the evolution process of natural selection to find (search) the optimal solution of a desired problem. GA works with a population of chromosome, each represents a potential solution and fitness value to a desired problem. Chromosomes with the higher fitness have a better chance to progress their information to the next generation through the crossover and gene mutation process. If a GA is designed well, the population will finally converge to an optimal solution. In this paper, Fourier descriptors are selected as the fitting function and its coefficients become the chromosomes of the GA. Results show that the proposed algorithm can handle complex (e.g. separated curves, separated points, partially occurred or partially missed (opened contour)) symmetry figures.

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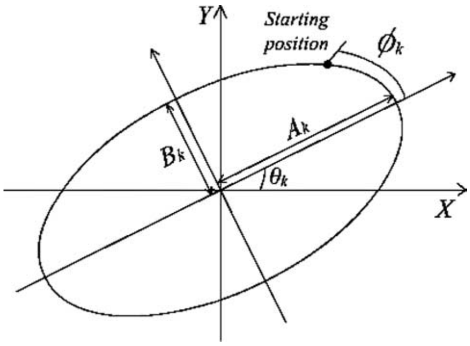

$$\begin{bmatrix} X_k(t) \\ Y_k(t) \end{bmatrix} = \begin{bmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{bmatrix} \begin{bmatrix} A_k & 0 \\ 0 & B_k \end{bmatrix} \begin{bmatrix} \cos\phi_k & -\sin\phi_k \\ \sin\phi_k & \cos\phi_k \end{bmatrix} \begin{bmatrix} \cos kt \\ \sin kt \end{bmatrix}$$

Fig. 1. (a) The idea of arc length parameterization. (b) Graphical meaning of Fourier descriptors with maximum $k=2$. (c) Geometric meaning of the rotation angle θ_k and the starting phase ϕ_k of the kth harmonic ellipse.

2. Elliptic Fourier descriptors

The basic idea of elliptic Fourier descriptors is as follows. A closed curve may be represented by a periodic function of a continuous parameter t, Eqs. (1) or (2)

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \begin{bmatrix} \cos kt \\ \sin kt \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{bmatrix} \begin{bmatrix} A_k & 0 \\ 0 & B_k \end{bmatrix} \times \begin{bmatrix} \cos\phi_k & -\sin\phi_k \\ \sin\phi_k & \cos\phi_k \end{bmatrix} \begin{bmatrix} \cos kt \\ \sin kt \end{bmatrix} \tag{2}$$

where $t=0$ to 2π; $(X(t),Y(t))$ is the coordinate of the contour in the image planes; $k=1, 2,\ldots, \infty$ and $[a_k b_k c_k d_k]$ are the kth coefficients of the FD.

In using Fourier descriptors for pattern recognition, a curve representation must be normalized with respect to a desired transformation domain so that the FD's are invariant with respect to the specified domain [2–7]. Invariant of FDs for object recognition and symmetry detection under 2D and 3D (affine) transform have been widely investigated. However, the arc length parameterization method needs to trace (usually use chain code) a close contour for finding the perimeter L, this leads to difficulties in handle the following situation.

(1) Complex figure—difficult to find the correct perimeter L for the symmetry interpretation.

Fig. 8a shows an example with complex crossing boundary. These kinds of figures are difficult to identify the correct tracing direction in their crossing boundaries (e.g. the centre of the Fig. 8). Hence, it is difficult to find the correct perimeter L for the symmetric interpretation of these figures.

(2) Partially occurred object—cannot find the correct perimeter L for symmetry interpretation.

Fig. 9a shows an example. As the symmetric figure is occurred by an irregular shape, only the contour of the circular (Fig. 9b solution 1) interpretation has the chance to find out by the chain code tracing mechanism.

(3) Open contour (e.g. Fig. 10)—there is no perimeter for an open contour.

Fig. 10a shows an example. Unless we connect the separate curves together, there is no way to obtain the perimeter L.

This paper proposes a new strategy using a genetic-Fourier (GFD) algorithm for the extraction of rotational symmetry without using any model and contour tracing mechanism. The proposed method has the following advantages.

(1) It eliminates the arc length parameterization method (no need to trace boundary) so it can handle partially occurred contour and opened contour.
(2) It eliminates the image points' connection process so it can handle complex point pattern.
(3) It can interpreted multiple perceptions.

2.1. FD invariants of rotational symmetry

Recalling the Elliptic FDs invariants of rotational symmetry from [5].

Property 1: For the coefficients of the kth harmonic, if $k\neq nN_s+1$ and $k\neq nN_s-1$, then $a_k=b_k=c_k=d_k=0$ where $n=1, 2, 3,\ldots$

Property 2: For the coefficients of the kth harmonic, if $k=nN_s-1$, then $A_k=-B_k$; if $k=nN_s+1$, then $A_k=B_k$ i.e. $|a_k|=|d_k|$, $|b_k|=|c_k|$ and $\theta=\theta_1=\theta_k$ where $n=1, 2, 3,\ldots$

Fig. 2 illustrates the above properties with $N=4$, $(a_0, c_0)=(128, 128)$, $A_1=50$, $\theta_1=0$, $A_3=-8$ and $A_5=-15$

From Eqs. (1), (2) and properties 1 and 2, it can be obtained that

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \sum_{k=1}^{\infty} \begin{bmatrix} A_k & 0 \\ 0 & B_k \end{bmatrix} \begin{bmatrix} \cos(kt+\phi_k) \\ \sin(kt+\phi_k) \end{bmatrix}$$

where $x_0=a_0$, $y_0=c_0$, $\theta=\theta_1$

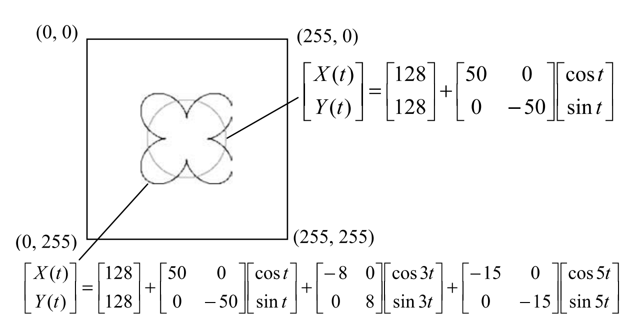

Fig. 2. Example showing properties 1 and 2 of elliptic Fourier descriptors.