Computer Vision and Image Understanding 117 (2013) 97–112

Contents lists available at SciVerse ScienceDirect

Computer Vision and Image Understanding

journal homepage: www.elsevier.com/locate/cviu

Unbiased extraction of lines with parabolic and Gaussian profiles ☆

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ARTICLE INFO

Article history:
Received 12 November 2010
Accepted 21 August 2012
Available online 8 September 2012

Keywords:
Feature extraction
Curvilinear structures
Lines
Scale-space
Low-level processing
Medical images

ABSTRACT

This paper presents an approach to extract curvilinear structures (lines) and their widths from two-dimensional images with high accuracy. Models for asymmetric parabolic and Gaussian line profiles are proposed. These types of lines occur frequently in applications. Scale-space descriptions of parabolic and Gaussian lines are derived in closed form. A detailed analysis of these scale-space descriptions shows that parabolic and Gaussian lines are biased more significantly than the well-known asymmetric bar-shaped lines by the partial derivatives of the Gaussian filters that are used to extract the lines. A bias function is constructed that relates the parameters of the lines to biased measurements that can be extracted from the image. It is shown that this bias function can be inverted. This is used to derive an algorithm to remove the bias from the line positions and widths. Examples on synthetic and real images show the high subpixel accuracy that can be achieved with the proposed algorithm. In particular, the line extractor is tested on a publicly available data set that includes manually labeled ground truth. The results on this data set show that very accurate results can be achieved on real data if the appropriate line model is used.

1. Introduction

The extraction of curvilinear structures, often simply called lines, has applications in many fields of science and technology. In the field of photogrammetry and remote sensing, it can be used in systems that extract roads from aerial and satellite images with different modalities, e.g., optical, synthetic aperture radar (SAR), and light detection and ranging (LIDAR) images [1–4], to extract road markings [3], to verify roads that are stored in geographic information systems (GISs) [5], to register maps to images [6], and even to extract buildings from SAR images, where lines occur as double reflections at concave dihedral corners on buildings [7]. Furthermore, line extraction can be used in the field of document analysis, e.g., to interpret engineering drawings [8]. Applications in the field of computer vision include 3D reconstruction using structured light [9–11] and stereo reconstruction [12,13]. In physics, line extraction can be used to detect gravitational waves in time–frequency diagrams [14,15]. The same techniques can also be used to detect sound events in pitch–time spectrograms of audio signals [16]. In the field of medical image analysis, applications of line extraction include the extraction of blood vessels for ophthalmic applications [17–19], for measurement of microcirculatory geometry [20], and for image registration [21,22], the detection of network patterns in skin lesions [23–25], the measurement of neurites [26,27] and neurite growth [28,29], and the detection of vesicle movements [30].

In all of the above applications, it is necessary to extract the line positions and widths with high accuracy. However, it is well known that the smoothing that must be used to extract features like lines and edges from an image inherently leads to biased extraction results. This effect was first studied qualitatively in the context of edge extraction [31,32]. The bias inherent in line extraction is analyzed quantitatively in [33]: a line detection algorithm based on differential geometry is described. Furthermore, an asymmetric bar-shaped line profile is proposed and its scale-space behavior is analyzed in detail. The analysis shows that the line positions are severely biased whenever the line is asymmetric and that the line widths are always biased. An algorithm to remove the bias from the extraction results is proposed, which results in very accurate line positions and widths. The algorithm is extended in [34,35] to handle asymmetric staircase lines (lines for which the line has a gray value that lies between the gray values of the background on the left and right side of the line) and to complete missing junctions. Furthermore, [35] extends the algorithm to extract lines from multispectral images, e.g., RGB color images.

Several other approaches to line detection have been proposed. The algorithms that have been published until 1998 are reviewed in detail in [33–35]. Therefore, only approaches that have been published since 1998 will be reviewed here.

A line detection algorithm that uses multiple-orientation Gabor filtering is proposed in [36]. The approach assumes lines to have a Gaussian profile. Lines are extracted by convolving the image with

☆ This paper has been recommended for acceptance by J.K. Udupa.

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http://dx.doi.org/10.1016/j.cviu.2012.08.007

oriented filters that are applied in 1° steps. Because of the very large number of filters that are employed, the approach is very slow. Furthermore, the algorithm extracts a pixel-precise region that corresponds to the line and not the line position and width.

An algorithm that uses the same line extraction principle as [33] is proposed in [37,38] (an earlier version of the algorithm is described in [39]). The approach uses the function of the 1D Deriche smoothing filter [40] as the model line profile. It then uses the same principles that were used to derive the Canny edge detector [41] to compute an optimal line detection filter. Because of the choice of the model line profile, the optimal line detection filters turn out to be the first and second derivatives of the Deriche smoothing filter. The approach does not consider the effect of the Deriche filters on the line positions and widths. Furthermore, the approach does not consider asymmetrical line profiles. Since any kind of smoothing leads to biased extraction results, the output of this line detector is also biased, and thus not optimal.

A multi-scale ridge detector that is based on the smoothed structure tensor is described in [42]. This algorithm uses the same principles for line detection that were first described in [43] and extends them by a multi-scale approach. The lines extracted with this approach are only pixel-precise. Furthermore, the line width is not extracted. More importantly, the bias for asymmetrical lines is not modeled. A little experimentation shows that this kind of line detector will return significantly biased results for the line positions for asymmetric lines (and for the line widths if this kind of detector were extended to extract the line width).

A line extractor that defines the line by maximizing the dissimilarity between features within the line to features on both sides of the line is described in [13]. The criterion is that the line is different from both sides of the line in frequency space and that both sides of the line are similar in frequency space. This facilitates the extraction of textured lines. The approach is iterated for different distances between the line profiles, and the optimum response is used. Hence, lines of different widths can be extracted. The approach is very time-consuming. The accuracy of the line widths depends on the discretization of the optimization search, which is relatively coarse. Errors in the extracted line positions and widths of more than one pixel on average are reported in [13].

Finally, an approach to line extraction based on nonlinear filtering is proposed in [44]. The filter uses circular masks, within which the pixels are weighted according to how similar their gray value is to the gray value at the center of the mask. If the Gaussian-weighted sum of these evaluations is below a threshold, the pixel is labeled as a line pixel. This approach is basically a region segmentation algorithm, i.e., it returns the areas in an image that correspond to a line. The line position and width are not extracted explicitly.

While the asymmetric bar-shaped line profile proposed in [33] is the correct model for many applications, in some of the above applications, a more suitable line profile is desirable to achieve higher accuracies. For example, in applications in which radiation is transmitted through translucent tubular objects, the bar-shaped line model is suboptimal. This class of applications includes, for example, X-ray or optical images of blood vessels.

One type of line profile suitable for these applications is a parabolic line profile. This type of profile is appropriate if the lines themselves have crisp boundaries and the optics and sensor do not cause a significant blurring of the image. This kind of line profile has been used in several applications, e.g., in [45] to extract neurons in confocal microscopic images and in the structured light system described in [9]. The parabolic line profile is also an excellent approximation to an elliptical profile [46] and to the profile described in [47], which models the blood column and vessel wall explicitly, determines their absorption coefficients, and inserts them into the Beer–Lambert law.

A second type of line profile that can be used in these applications is a Gaussian line profile. This kind of profile is appropriate if the lines themselves appear blurred, e.g., because they are buried in a material that causes scatter [48] or because the optics or sensor cause significant blurring [46,49,48]. Gaussian profiles have been used in several applications, e.g., to extract blood vessels in retinal images [50] or in coronary angiograms [36]. The Gaussian line profile has been shown to be an appropriate model for extracting blood vessels from conjunctival images [48] and from retinal images [19]. It is also an excellent approximation to the smoothed elliptical profile used in [46,49,48].

Fig. 1 shows that the parabolic and Gaussian line models provide a very good match for the profiles that occur in several real applications.

This paper is organized as follows: Section 2 briefly describes the line detection algorithm. Section 3 defines the parabolic and Gaussian line models. The scale-space behavior of the line models is analyzed in Section 4. Section 5 describes the bias removal algorithm. Examples are given in Section 6. Finally, Section 7 concludes the paper.

2. Line detection algorithm

To make this paper self-contained, the underlying line detection algorithm will be described briefly. A detailed description can be found in [33,34].

For the derivation of the bias removal in this paper, it is useful to study the definition of lines in one-dimensional images, i.e., gray value profiles, first. As discussed below, lines in two-dimensional images are modeled as having one of the characteristic gray value profiles across the line, i.e., perpendicular to the line. Let us call the gray value profile $f(x)$. Let us assume that the profile is the parabolic or Gaussian profile shown in Fig. 1. Then, lines are defined as points where $f'(x) = 0$, i.e., local maxima (for bright lines) or local minima (for dark lines) of the gray value profile. For real images, which contain noise, this criterion must be augmented with a criterion to select salient lines. This can be achieved with a threshold on $|f''(x)|$, i.e., by requiring $f''(x) \ll 0$ for bright lines and $f''(x) \gg 0$ for dark lines. Furthermore, since for real images the determination of the image derivatives is an inherently ill-posed problem, the above derivatives are estimated by convolving the image with the derivatives of a Gaussian kernel. It is well known that, under very general assumptions, the Gaussian kernel is the only kernel that makes the problem of estimating the derivatives of a noisy function well-posed [51]. The Gaussian kernel and its derivatives are given by:

$$g(x,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}, \tag{1}$$

$$g'(x,\sigma) = \frac{-x}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2}{2\sigma^2}}, \tag{2}$$

$$g''(x,\sigma) = \frac{x^2-\sigma^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2}{2\sigma^2}}. \tag{3}$$

In the analysis below, we will also need the integral of the Gaussian kernel:

$$\phi(x,\sigma) = \int_{-\infty}^{x} e^{-\frac{t^2}{2\sigma^2}} \mathrm{d}t. \tag{4}$$

By convolving the gray value profile $f(x)$ with the derivatives of the Gaussian kernel, a scale-space description of the profile is obtained:

$$r(x,\sigma) = g(x,\sigma) * f(x), \tag{5}$$

$$r'(x,\sigma) = g'(x,\sigma) * f(x), \tag{6}$$

$$r''(x,\sigma) = g''(x,\sigma) * f(x). \tag{7}$$