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Quantitative orness for lattice OWA operators

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ABSTRACT

This paper deals with OWA (ordered weighted average) operators defined on any complete lattice endowed with a t-norm and a t-conorm and satisfying a certain finiteness local condition. A parametrization of these operators is suggested by introducing a quantitative orness measure for each OWA operator, based on its proximity to the OR operator. The meaning of this measure is analyzed for some concrete OWA operators used in color image reduction, as well as for some OWA operators used in a medical decision making process.

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1. Introduction

Ordered weighted average (OWA) operators were introduced by Yager in [1] in order to obtain a global value by aggregating several data on the real interval [0, 1]. Unlike for other weighted average operators, the weight associated to each datum in an OWA operator only depends on the position it takes in the descending arrangement of the data. Hence Yager's OWA operators are symmetric, i.e., the global value they provide does not depend on the order that the data are considered.

In addition, they form a wide family of aggregation functions situated between the AND operator, which provides the minimum of the given values, and the OR operator, which gives the maximum of them. For this reason, OWA operators are commonly used in data fusion or multicriteria decision making [2–7].

The orness of an OWA operator was proposed by Yager in [8] as a measure of its proximity to the OR operator. This way, the orness provides a classification of all the OWA operators defined on the real interval [0, 1], giving the OR operator the maximum value 1 and the AND operator the minimum value 0. This classification is of great help in order to choose the weighting vector in each practical application. In other words, the orness of each OWA operator provides information about the influence that each concrete choice of weighting vector will have in the aggregation result (see also [9–11]).

In some practical applications, the data to aggregate are not either numerical or linearly ordered [12–16]. This is the case, for example, of fuzzy sets and some of their extensions [17]. Moreover, some medical decision making problems require to merge opinions from different experts, such as *aggressive surgery, conservative surgery, radiotherapy* or *chemotherapy*, which are not easily arranged. The lattice structure is a suitable way to model the interrelations of these options. A lattice structure also occurs in image processing in RGB system, where each pixel is represented by a tern consisting of three numerical components. In spite of its numerical nature, the set of all of these terns, not totally ordered, forms a complete lattice.

As it had been done with other aggregation functions [18], OWA operators were generalized in [19] from the real unit interval to a general complete lattice endowed with a t-norm T and a t-conorm S, whenever the weighting vector satisfied a distributivity condition with respect to T and S. Following the way paved by Yager, a qualitative parametrization of OWA operators defined on an arbitrary finite lattice based on their proximity to the OR operator was given in [20]. In addition, the semantic meaning of this qualitative measure was explained by means of its application to some examples taken from both image processing and medical decision making.

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In this paper, a quantitative orness measure is proposed in order to get a new parametrization of OWA operators defined on any complete lattice L endowed with a t-norm T and a t-conorm S and satisfying some finiteness local condition. If we defined it as an aggregation of the weights $(\alpha_1, \ldots, \alpha_n) \in L^n$ as Yager does in the real case, where $\text{orness}(F_\alpha) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)\alpha_i$, we would obtain an element of the lattice L as a result instead of a number.

So, we propose in this paper to define firstly a qualitative quantifier $Q: \{0, 1, \ldots, n\} \to L$ by means of $Q_\alpha(0) = 0_L$ and $Q_\alpha(k) = S[\alpha_1, \ldots, \alpha_k]$ for $1 \le k \le n$ and then to aggregate certain numerical distances $M(k)$ between $Q_\alpha(k)$ and $Q_\alpha(k-1)$ to get a quantitative orness measure:

$$\text{orness}(F_\alpha) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)M(i).$$

In the case that $L = [0, 1]$, the distance between each $Q_\alpha(k) = \alpha_1 + \cdots + \alpha_k$ and $Q_\alpha(k-1) = \alpha_1 + \cdots + \alpha_{k-1}$ is exactly α_k. So, our concept of orness is a generalization of Yager's one.

We prove that this concept satisfies the properties imposed in [11] to any orness measure in an axiomatic framework. In the particular case of a distributive complete lattice, it is shown that the OWA operator can be retrieved from the quantifier associated to its weighting vector.

The advantage of a quantitative orness over a qualitative one is the possibility of dividing OWA operators into OR-like ones, those with an orness greater (or equal) than 0.5 and AND-like ones, those whose orness is less than 0.5. In contrast, Example 5.1 shows that a qualitative orness gives more information than the quantitative one about the meaning of choosing some weighting vector or another in a decision making process.

Finally, the new parametrization of OWA operators is applied to analyze the same examples that were studied in [20] by means of the qualitative orness measure. The first one deals with a decision making problem about the kind of medical treatment to use with a breast cancer patient. The second one consists of an image reduction algorithm in the RGB color scheme performed by means of several lattice OWA operators. Of course, different OWA operators provide different solutions to each problem, which are analyzed on the basis of the orness of the OWA operators. Moreover, we also analyze the effect of OWA operators on images altered by certain type of impulsive noise.

The paper is organized as follows. Section 2 is devoted to bring together some preliminary concepts and results about OWA operators defined on a complete lattice. Section 3 suggests a quantitative orness for OWA operators defined on complete lattices. Section 4 focuses on the case of a complete distributive lattice, showing that the OWA operator can be retrieved in this case from the quantifier associated to its weighting vector. Finally, Section 5 applies the orness measure to analyze a decision making problem and Section 6 studies the meaning of the orness in an application to a problem of color image reduction. A final section of conclusions and further research closes the paper.

2. Preliminaries

Throughout this paper $(L, \le_L)$ will denote a complete lattice, i.e., a partially ordered set in which all subsets have both a supremum and an infimum. 0_L and 1_L will respectively stand for the least and the greatest elements of L. A lattice L is said to be complemented if for each $a \in L$ there exists some $b \in L$ such that $a \wedge b = 0_L$ and $a \vee b = 1_L$. A subset M of L is called a sublattice of $(L, \le_L)$ if whenever $a, b \in M$, then both $a \wedge b$ and $a \vee b$ belong to M.

Definition 2.1 (see [21]). A map $T: L \times L \to L$ is said to be a t-norm [resp. t-conorm] on $(L, \le_L)$ if it is commutative, associative, increasing in each component and has a neutral element 1_L [resp. 0_L].

Notation: For any $n > 2$, $S(a_1, \ldots, a_n)$ will denote $S[\ldots(S(S(a_1, a_2), a_3), \ldots a_{n-1}), a_n]$. Note that, for any permutation σ of the elements $1, \ldots, n$,

$S(a_1, \ldots, a_n) = S(a_{\sigma(1)}, \ldots, a_{\sigma(n)})$.

Throughout this paper $(L, \le_L, T, S)$ will denote a complete lattice endowed with a t-norm T and a t-conorm S. As usual, L^n will denote the cartesian product $L \times \cdots \times L$ and L^{I_n} will stand for the set of all the n-ary lattice intervals $[a_1, \ldots, a_n]$ with $a_1 \le_L \cdots \le_L a_n$ contained in L.

Recall that an n-ary aggregation function is a function M: $L^n \to L$ such that:

(i) $M(a_1, \ldots, a_n) \le_L M(a'_1, \ldots, a'_n)$ whenever $a_i \le_L a'_i$ for $1 \le i \le n$.
(ii) $M(0_L, \ldots, 0_L) = 0_L$ and $M(1_L, \ldots, 1_L) = 1_L$.

It is said to be *idempotent* if $M(a, \ldots, a) = a$ for every $a \in L$ and it is called *symmetric* if, for every permutation σ of the set $\{1, \ldots, n\}$, $M(a_1, \ldots, a_n) = M(a_{\sigma(1)}, \ldots, a_{\sigma(n)})$.

A wide family of both symmetric and idempotent aggregation functions was introduced by Yager in [1] on the lattice $L = [0, 1]$, the real unit interval:

Definition 2.2 (Yager [1]). Let $\alpha = (\alpha_1, \ldots, \alpha_n) \in [0, 1]^n$ be a weighting vector with $\alpha_1 + \cdots + \alpha_n = 1$. An n-ary *ordered weighted average operator* or *OWA operator* is a map F_α: $[0, 1]^n \to [0, 1]$ given by

$F_\alpha(a_1, \ldots, a_n) = \alpha_1 b_1 + \cdots + \alpha_n b_n$,

where $(b_1, \ldots, b_n)$ is a rearrangement of $(a_1, \ldots, a_n)$ satisfying that $b_1 \ge \cdots \ge b_n$.

It is easy to check that OWA operators form a family of aggregation functions bounded between the AND-operator or minimum, given by the weighting vector $\alpha = (0, \ldots, 0, 1)$,

$F_\alpha(a_1, \ldots, a_n) = a_1 \wedge \cdots \wedge a_n$ for any $(a_1, \ldots, a_n) \in [0, 1]^n$

and the OR-operator or maximum, given by the weighting vector $\alpha = (1, 0, \ldots, 0)$,

$F_\alpha(a_1, \ldots, a_n) = a_1 \vee \cdots \vee a_n$ for any $(a_1, \ldots, a_n) \in [0, 1]^n$.

With the purpose of classifying these operators, Yager introduced in [8] an orness measure for each OWA operator F_α, which depends only on the weighting vector $\alpha = (\alpha_1, \ldots, \alpha_n)$, in the following way:

$$\text{orness}(F_\alpha) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)\alpha_i = F_\alpha\left(1, \frac{n-2}{n-1}, \ldots, \frac{1}{n-1}, 0\right). \quad (1)$$

It is easy to check that the orness of each operator is a real value situated between 0, corresponding to the AND-operator, and 1, corresponding to the OR-operator. In general, the orness is a measure of the proximity of each OWA operator to the OR-operator. For instance, the orness of the arithmetic mean, provided by the weighting vector $(1/n, \ldots, 1/n)$, is equal to 1/2.

In addition, Yager defines, for any weighting vector $\alpha = (\alpha_1, \ldots, \alpha_n) \in [0, 1]^n$, a function $Q_\alpha: \{0, 1, \ldots, n\} \to [0, 1]$, called *quantifier* by means of:

$$Q_\alpha(k) = \begin{cases} 0 & \text{if } k = 0 \\ \alpha_1 + \cdots + \alpha_k & \text{otherwise} \end{cases} \quad (2)$$

Notice that Q_α is a monotonically increasing function. Moreover, given a monotonically increasing function $Q: \{0, 1, \ldots, n\} \to [0, 1]$ with $Q(0) = 0$ and $Q(n) = 1$, then there exists a unique weighting vector $\alpha = (\alpha_1, \ldots, \alpha_n) \in [0, 1]^n$ with $Q_\alpha = Q$. Indeed, for any $k = 1, \ldots, n$, put $\alpha_k = Q(k) - Q(k-1)$ and check that $Q_\alpha = Q$.

In [19] n-ary ordered weighted average (OWA) operators are extended to any complete lattice endowed with a t-norm T and a t-conorm S whenever the weighting vector satisfies some distributivity condition.

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