Information Fusion 30 (2016) 91–102

Contents lists available at ScienceDirect

Information Fusion

journal homepage: www.elsevier.com/locate/inffus

Gaussian-consensus filter for nonlinear systems with randomly delayed measurements in sensor networks

Yanbo Yang[a,b,*], Yan Liang[a,b], Quan Pan[a,b], Yuemei Qin[a,b], Xiaoxu Wang[a,b]

[a] School of Automation, Northwestern Polytechnical University, Xi'an 710072, PR China
[b] Key Laboratory of Information Fusion Technology, Ministry of Education, Xi'an 710072, PR China

ARTICLE INFO

Article history:
Received 14 February 2015
Revised 12 October 2015
Accepted 9 December 2015
Available online 17 December 2015

Keywords:
Gaussian approximated filter
Sensor networks
Consensus filter
Randomly delayed measurements
Markov chain

ABSTRACT

This paper presents the decentralized state estimation problem of discrete-time nonlinear systems with randomly delayed measurements in sensor networks. In this problem, measurement data from the sensor network is sent to a remote processing network via data transmission network, with random measurement delays obeying a Markov chain. Here, we present the Gaussian-consensus filter (GCF) to pursue a tradeoff between estimate accuracy and computing time. It includes a novel Gaussian approximated filter with estimated delay probability (GEDPF) online in the sense of minimizing the estimate error covariance in each local processing unit (PU), and a consensus strategy among PUs in processing network to give a fast decentralized fusion. A numerical example with different measurement delays is simulated to validate the proposed method.

1. Introduction

Considerable research has been undertaken in the field of estimation theory in relation to the discrete-time nonlinear systems over the past several decades due to its widespread applications in process control [1,2], signal processing [3], fault detection and isolation [4], integrated navigation [5] and target tracking [6,7]. In general, the minimum-mean-square-error estimator for nonlinear systems is almost always intractable in the Bayesian view [8], and hence much attention has been paid on approximation strategies for designing cost-effective estimators. One strategy is the function approximation, i.e., nonlinear dynamic/measurement functions are replaced by piece-wise time-varying linear functions, including the extended Kalman filter [8] via Taylor expansion, the central difference filter [9] based on derivative operation and the divided difference filter [10] based on interpolation polynomial. Generally speaking, function-approximation estimators are computation-effective but sensitive to linearized errors or differential operations. An alternative strategy is the density approximation, i.e., the conditional state probability density function is represented as a Gaussian or Gaussian mixture distribution [11,12]. This density-approximation strategy results in the integrated framework of analytical computation and numerical integration. Through choosing different numerical integration schemes, the computation burden and estimation accuracy can be balanced. Up to now, the resultant filters are also called Gaussian Approximated Filters (GAFs) including the unscented Kalman filter (UKF) [13] based on unscented transformation, the Gauss-Hermite Filter (GHF) [14] based on Gaussian-Hermite quadrature rule, the new quadrature Kalman filter [15] based on statistical linear regression, the square-root quadrature Kalman filter [16] based on matrix triangularization, the cubature Kalman filter (CKF) [17] based on spherical-radial cubature rule. In general, all these GAFs are limited in the delay-free scope, i.e., the measurement arrives on time.

However, in many actual situations, measurements may arrive at the data processing center with random delays. For example, in a networked multi-sensor remote-sensing system, sensors may be geographically far away from estimators/controllers and hence the random measurement delay is definitely inevitable due to limited-capability data transmission or additional routing. It motivates the research on state estimation with randomly delayed measurements [18–20]. A stochastic extended Kalman filter was presented for interconnected power systems with partially or totally delayed measurements [21]. The extended and unscented filters were proposed for a class of nonlinear discrete-time stochastic systems with one-step Bernoulli random measurement delay [22]. Moreover, via Gaussian approximations of the one-step posterior predictive probability density functions of state and delayed measurement, a novel GAF was proposed for a nonlinear stochastic systems with one-step randomly Bernoulli delayed measurements [23]. Similarly, the corresponding Gaussian smoother was also derived [24]. A new

* Corresponding author at: School of Automation, Northwestern Polytechnical University, Xi'an 710072, PR China. Tel.: +8615829366908.
E-mail address: yangyanbo9@mail.nwpu.edu.cn, yangyb8911@gmail.com (Y. Yang).

http://dx.doi.org/10.1016/j.inffus.2015.12.003
1566-2535/

unscented filtering was presented for a class of nonlinear stochastic systems with random measurement delays less than three time instants, and this result was extended to deal with multi-step Bernoulli delayed measurements [25]. In general, all these methods have the common assumption that the delay should be Bernoulli distributed with known probability. However, such an assumption may not hold since, in network-based data transmissions, time delays typically occur in a batch mode, and the transition from one mode to another may obey a certain probability distribution [26]. Meanwhile, random delays in a networked system may exhibit the feature that the occurrence of current delay depends on its previous delay [27]. Therefore, a seemingly more realistic assumption is that the switching between random delays at adjacent instants abides by a Markov chain, which includes the Bernoulli assumption as a special case [28]. Furthermore, above discussed methods for nonlinear systems with randomly delayed measurements all require centralized processing, i.e., measurements, maybe from different sources, should be collected together and then processed at a fusion center. Actually, the delay occurs inevitably by computation sources being far away from sensors in remote sensing or control. At the same time, many computing units may be deployed, constituting a network in order to accommodate massive sensor data processing. Therefore, it is necessary to develop a decentralized processing and fusion method.

So far, to the best of authors' knowledge, the state estimation and decentralized fusion for nonlinear systems with random measurement delays obeying a Markov chain in sensor networks has not been investigated, which motivates us to formulate this new problem. In the considered problem, sensor data collected by a sensor network is sent to a processing network via transmission network. The data transmission incurs the random delay and hence leads to the existence of multi-mode uncertainties and stochastic parameters which are coupled with system nonlinearity and stochastic noises.

The technical contributions of this paper are as follows. Firstly, a Markov process is utilized to depict the random measurement delay in each processing unit (PU). Such modeling is more general and practical, by the fact that the Markov process includes the Bernoulli process which is a widely-accepted random delay model in existing researches [22,23,25] as a special case and can exploit the relationship of delays at adjacent sampling instants [26,27]. Secondly, a novel and generalized GAF with estimated delay probability (GEDPF) is derived for local state estimation in each PU. Here, the delay probability is unknown and even time-varying instead of being known in [22,23,25]. Meanwhile, the derived GEDPF will be degraded to the common GAF in [14] if there is no delay, or the GAF in [23] if the delay is one-step Bernoulli distributed with known probability. Thirdly, the derived GEDPF is based on lower-dimensional state augmentation, which leads lower-dimensional matrix calculations and Gaussian approximations, compared with the unscented filtering with multi-step random delays in [25]. Fourthly, combined with consensus strategy, the proposed Gaussian-consensus filter (GCF) gives a decentralized fusion implementation with random measurement delays, which has never been investigated for existing filters with delayed measurements in [22,23,25] or consensus filters for networked sensors [29–31].

The rest of this paper is organized as follows. The problem formulation is presented in Section 2. The GEDPF is derived for nonlinear systems with randomly delayed measurements and the GCF is further presented in the processing network in Section 3. A numerical example is simulated in Section 4 to validate the proposed method. Finally, the conclusion is supplied in Section 5. All proofs are presented in the Appendix.

Notation: Throughout this paper, superscripts "-1" and "T" represent the inverse and transpose operation of matrix, respectively. I and O denote the identity and zero matrices of appropriate dimensions, respectively. The symbol " $:=$ " means definition. $E(\cdot)$ and $\mathrm{cov}(\cdot)$ denote mathematical expectation and covariance calculation, respectively. $(\cdot)$ denotes the same content as that in the previous parentheses. The Gaussian distribution $N(\wp_\rho;\hat{\wp}_{\rho|k},P^{\wp\wp}_{\rho|k})$ is denoted by $G_{\rho|k}(\wp)$. $\|\cdot\|$ denotes the 2-norm of a vector.

2. Problem formulation

Consider the problem of multi-sensor state estimation for a nonlinear dynamic process as shown in Fig. 1. The whole system contains a sensor network, a transmission network and a processing network. Here, the sensor network contains a large number of clustered sensor nodes to detect and collect state information. Then, the transmission network is in charge of sending sensor measurements to buffers, and measurements are subjected to random delays in transmission. The processing network is constituted by many PUs, and each PU obtains a localized estimate based on the newly-received delayed measurements from its corresponding sensor cluster and exchanges its estimate with its neighbors for estimation consensus.

Here, the random delay may exhibit the feature that the occurrence of current delay depends on its previous delay [27], or the transition from one delayed step to another may obey a certain probability distribution [26]. Thus, a practically random process to describe this delay is the first-order Markov process.

Motivated by this, we formulate a new estimation problem as follows:

$$\text{nonlinear dynamical process}: x_{k+1} = f_k(x_k) + w_k, \tag{1}$$

the sampled measurement in a sensor node :

$$z_{k+1} = h_{k+1}(x_{k+1}) + v_{k+1}, \tag{2}$$

where w_k and v_k are uncorrelated zero-mean Gaussian, white noises satisfying $E(w_k w_t^T) = Q_k\delta_{kt}$ and $E(v_k v_t^T) = R_k\delta_{kt}$, respectively, with δ_{kt} being the Kronecker delta function. Meanwhile, the initial state is a Gaussian random vector with mean x_0 and covariance P_0, being uncorrelated with w_k and v_k.

In an ideal condition, a sensor measurement is transmitted to the corresponding buffer and taken out in time for local state estimation in processing network. However, sensors may be far away from buffers or buffers may be far away from estimators, and the measurement may arrive after the corresponding state estimation in several steps. Thus, random measurement delay due to data transmission and relay or network congestion is definitely inevitable. Consider the case that any induced latency from a sensor to an estimator in the mth PU is not more than s_m-step sampling period, we will obtain the following measurement output from the corresponding buffer:

the received meaurement in the mth PU :

$$y^m_{k+1} = \sum_{j=0}^{\min(k+1,s_m)} \gamma^m_{j,k+1} z^m_{k+1-j}, \tag{3}$$

where z^m_{k+1-j} is the sampled measurement from the corresponding sensor node. s_m denotes the maximum delay step in the mth PU. $\gamma^m_{j,k+1}$ is a 0-1 binary random variable obeying a discrete-time Markov chain with the switching probability:

$$P\left\{\gamma^m_{j,k+1} = 1 | \gamma^m_{i,k} = 1\right\} = \lambda^m_{ij,k+1}, \quad i,j = 0,1,\cdots,s_m, \quad m = 1,\cdots,M. \tag{4}$$

Meanwhile, assume that the newly-received measurement y_{k+1} is only from one sampling instant, and hence the following constraint