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A position and perspective analysis of hesitant fuzzy sets on information fusion in decision making. Towards high quality progress

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ABSTRACT

The necessity of dealing with uncertainty in real world problems has been a long-term research challenge which has originated different methodologies and theories. Recently, the concept of Hesitant Fuzzy Sets (HFSs) has been introduced to model the uncertainty that often appears when it is necessary to establish the membership degree of an element and there are some possible values that make to hesitate about which one would be the right one. Many researchers have paid attention on this concept who have proposed diverse extensions, relationships with other types of fuzzy sets, different types of operators to compute with this type of information, applications on information fusion and decision-making, etc.

Nevertheless, some of these proposals are questionable, because they are straightforward extensions of previous works or they do not use the concept of HFSs in a suitable way. Therefore, this position paper studies the necessity of HFSs and provides a discussion about current proposals including a guideline that the proposals should follow and some challenges of HFSs.

1. Introduction

Uncertainty has been a long-term matter of research in many different fields from mathematics to health, management, engineering and so on [8]. Many different theories and methodologies have been developed in the literature to represent and manage different types of uncertainty in such fields. Recently, it was introduced the concept of Hesitant Fuzzy Sets (HFSs) [41,43] whose main aim is to model the uncertainty produced by the human beings *doubt* when eliciting information. A similar concept but with different operations and aim was introduced by Grattan-Guinness in [16].

In spite of the youth of HFSs, they have attracted the attention of a great number of researchers with hundreds of papers since its publication. The growing interest in HFSs has led to the introduction and analysis of multiple operations and properties on HFSs. Quantitative and qualitative extensions have been developed together their applications to real world problems, mainly on information fusion and decision-making (some of them can be found in the recent review [35]).

However, some of these proposals can be debatable either because of their marginal novelty and interest, they are mere straightforward developments of previous works, because they do not consider the premise that HFSs model uncertainty coming from *doubt* or because their applicability to real world problems is unlikely.

Therefore, this position paper (developed by several researchers who have had an important role in the development of the HFS theory, its extensions and applications) tries to present an analysis of

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the need of HFSs and afterwards different discussions about different topics on HFSs are carried out. They analyze the context of each topic to establish a discussion about current proposals, provide a common-sense point of view that HFS proposals should follow, and conclude with challenges of HFSs in different related issues.

The remaining sections of this paper are set up as follows: Section 2 reviews and fixes basic definitions and notations. Section 3 provides a clear and depth view about why to propose HFSs. Section 4 analyzes the foundations for HFSs. Section 5 shows the relationships of HFSs with other types of fuzzy sets. Section 6 discusses the HFS extensions. Section 7 presents a view about decision-making with HFSs. Section 8 provides an analysis on the aggregation operators for HFSs. Section 9 provides a discussion about similarity, entropy and distance measures with HFSs. Section 10 analyzes the consistency in HFSs and in Section 11 an analysis of consensus reaching processes in group decision-making with HFSs is introduced. Finally, Section 12 provides some concluding remarks about the results that can be abstracted from this position paper.

2. HFSs: basic definitions and notations

HFSs are a recent extension of fuzzy sets that models the uncertainty provoked by the hesitation that might appear when it is necessary to assign the membership degree of an element to a fuzzy set [41].

This section revises some basic concepts and operations about HFSs and clarifies the notations about such concepts.

2.1. Concepts

A HFS is defined in terms of a function that returns a set of membership values for each element in the domain.

Definition 1 [41]**.** Let X be a reference set. A HFS on X is a function $\mathfrak{h}$ that returns a non-empty subset of values in [0,1]:

$$\mathfrak{h} : X \rightarrow \wp([0,1]) \tag{1}$$

A HFS can be also constructed from a set of fuzzy sets.

Definition 2 [41]**.** Let $M = \{\mu_1, \ldots, \mu_n\}$ be a set of n membership functions. A HFS associated to M, $\mathfrak{h}_M$, is defined as follows:

$$\begin{aligned} &\mathfrak{h}_M : X \rightarrow \wp([0,1]) \\ &\mathfrak{h}_M(x) = \bigcup_{i=1}^{n} \{\mu_i(x)\}, \end{aligned} \tag{2}$$

where $x \in X$.

Xia and Xu [49] called $h(x)$ a hesitant fuzzy element (HFE). Note that a HFE is a set of values in [0,1], and a HFS is a set of HFEs, one for each element in the reference set. That is, if $h(x)$ is the HFE associated to x then $\cup_{x \in X} h(x)$ is a HFS.

Therefore, a HFS is a set of subsets in the interval [0,1], one set for each element of the reference set X and a HFE is one of such sets.

Recently, a particular case of HFS so-called Typical Hesitant Fuzzy Set [4], which takes into account some constraints, has been proposed.

Definition 3 [4]**.** Let $\mathbb{H} \subseteq \wp([0,1])$ be the set of all finite non-empty subsets of the interval [0,1], and let X be a non-empty set. A Typical Hesitant Fuzzy Set (THFS) A on X is given by a mapping $h_A : X \rightarrow \mathbb{H}$.

Each $h_A(x) \in \mathbb{H}$ is called a Typical Hesitant Fuzzy Element (THFE) of $\mathbb{H}$.

In real world problems, it is common to carry out computations with the information to obtain results. To do this, an extension principle to export operations on fuzzy sets for HFSs was introduced.

Definition 4 [43]**.** Let $E = \{H_1, \ldots, H_n\}$ be a set of n HFSs and Θ a function, Θ: $[0, 1]^n \rightarrow [0, 1]$, thus the function Θ on fuzzy sets is exported to HFS as follows:

$$\Theta_E = \bigcup_{\gamma \in H_1(x) \times \cdots \times H_n(x)} \{\Theta(\gamma)\} \tag{3}$$

Note that the properties on Θ lead to related properties on Θ_E. For example, commutativity and associativity of Θ lead to commutativity and associativity of Θ_E.

2.2. Basic operations

Several basic operations to deal with HFEs were defined in [41,49].

Definition 5 [41]**.** Given a HFE, h, its lower and upper bounds are:

$$h^- = inf\{\gamma | \gamma \in h\} \tag{4}$$

$$h^+ = sup\{\gamma | \gamma \in h\} \tag{5}$$

Definition 6 [41]**.** Let h be a HFE, its complement is defined as:

$$h^c = \bigcup_{\gamma \in h} \{1 - \gamma\} \tag{6}$$

Definition 7 [49]**.** Let h_1 and h_2 be two HFEs, their union is defined as follows:

$$h_1 \cup h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{max\{\gamma_1, \gamma_2\}\} \tag{7}$$

Definition 8 [49]**.** Let h_1 and h_2 be two HFEs, their intersection is defined as follows:

$$h_1 \cap h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{min\{\gamma_1, \gamma_2\}\} \tag{8}$$

A complete study of the state of the art, properties, extensions, operators and applications can be found in [35].

3. On why we introduced HFSs

HFSs were introduced in [41] as an extension of fuzzy sets. The basic idea was to model the case in which instead of a single membership degree, human beings hesitate among a set of membership degrees and they need to represent such a hesitation/doubt. This set of membership degrees is typically considered finite.

Torra [41] justified HFSs when it is difficult to establish a single value for the membership degree, not because there is a margin of error, or some possibility distribution on the possible values, but because there is a doubt among a set of possible values. An example is given in which there are several experts and each of them gives a different degree (see e.g. “some experts have only assigned such a small and finite set” in [41]). Similar scenarios are considered in [43], where focus is given to decision-making problems. Another possible example could be when there are several memberships, and all of them are considered to define a HFS (see Eq. (2)).

All these examples have in common that human beings are able to elicit or compute different degrees of membership to a set for a given object. Then, instead of using these degrees to build an interval, a type-2 fuzzy set, or even a confidence interval for the membership degree, all these degrees are kept to process them. A HFS permits to model these situations, providing a way to operate on the sets and postpone (if this is really needed) the aggregation or transformation of the values in the set into a single one.

It should be highlighted that a HFS also permits to deal with the case that different elements of the reference set have a distinct number of membership degrees.

HFSs are related to other extensions of fuzzy sets. Some of them were already established (see Section 5) and in [41] it was proved that