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The fusion process of interval opinions based on the dynamic bounded confidence

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ABSTRACT

In this paper, we propose a novel opinion dynamics model that is based on bounded confidence and termed interval opinion dynamics with the dynamic bounded confidence. In this opinion dynamics model, the agents express their opinions in numerical intervals, and the bounded confidences vary in a specified interval as time varies (i.e., dynamic bounded confidence). Based on several theoretical analyses of the proposed opinion dynamics, we propose conditions that are sufficient to form a consensus or fragmentations among the agents. Moreover, we also design several simulation experiments to investigate the effects of the dynamic bounded confidence and interval widths on the proposed opinion dynamics and to illustrate the differences between the proposed model and the original opinion dynamics with bounded confidence.

1. Introduction

Opinion dynamics is a fusion process of individual opinions that can be defined as a group of interacting agents who continuously update their opinions regarding the same issue based on the established fusion rules and reach a consensus (or fragmentation) in the final stage.

In opinion dynamics, the establishment of the fusion rule is the core problem [1]. Recently, some research results regarding the fusion rules have been reported, such as the voter model [2], the persuasiveness and supportiveness model [3], the bounded confidence model [4,5] and the Alexford model [6]. Among these existing models, the use of the bounded confidence model as the fusion rule has become a topic of intensive research in recent years.

The bounded confidence model assumes that the agents only communicate with the peers who hold similar opinions and tend to ignore the peers with sufficiently different opinions [7]. The earliest bounded confidence models were introduced independently by Hegselmann and Krause (HK model) [4] and Deffuant and Weisbuch (DW model) [5]. In the HK model, the agents synchronously update their opinions by averaging all of the opinions in their confidence set [4], and in the DW model, the agents adhere to pairwise-sequential updating mechanisms [5]. Based on the original HK and DW models, the following different types of bounded confidence models have been proposed: (i) The agent-based homogeneous models [8–13], (ii) The agent- based heterogeneous models [14–16], (iii) The density-based homogeneous models [17–20], and (iv) The density-based heterogeneous models [9,15,21].

Previous studies have significantly advanced the bounded confidence model. In this study, we propose the interval opinion dynamics with dynamic bounded confidence model. The study is motivated by the following two aspects:

(1) In the existing studies, each agent uses a crisp number to express his/her opinion, i.e., a crisp opinion. However, in the practical processes of opinion dynamics, the opinions of the agents often exhibit uncertainty. Thus, it is necessary to investigate the effects of the uncertain opinions on opinion dynamics. Generally, interval opinions are the most basic formats of uncertain opinions [22–24]. Thus, in the present study, the fusion of interval opinions will provide a foundation for investigating the effects of uncertain opinions on opinion dynamics.
(2) Bounded confidence is the basic assumption of the opinion dynamics problem. In the existing studies, there are two types of bounded confidences, i.e., homogeneous and heterogeneous bounded confidences. The former refers to cases in which all of the agents have a uniform bounded confidence. The latter refers to cases in which each agent has his/her own bounded confidence. However, in the practical processes of opinion dynamics, the bounded confidence will vary dynamically. For example, with deep interactions, the trust between the agents may be strengthened. In such situations, their bounded confidences will increase over time, whereas with increases in conflicts of opinions, the agents' bounded confidences decrease over time. Therefore, it is

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necessary to study the opinion dynamics with the assumption of dynamic bounded confidence.

The aim of this paper is to discuss the fusion process of interval opinions with dynamic bounded confidence. The remainder of this paper is arranged as follows. In Section 2, we introduce the HK model. Next, in Section 3, we propose the interval opinion dynamics model with dynamic bounded confidence. In section 4, we conduct the theoretical analyses for the proposed model. In the theoretical analyses, we provide the conditions that are sufficient for the formation of a consensus or fragmentations among the agents. Furthermore, in Section 5 we design some simulation experiments to investigate the effects of the dynamic bounded confidence and the interval widths on the proposed opinion dynamics and illustrate the differences between the proposed model and the original HK model. Finally, concluding remarks are presented in Section 6.

2. Preliminary: the HK model

In this section, we introduce the original HK model, which will provide a foundation for this study.

The fundamental difference between the DW and HK models is the number of agents that communicate, which is labelled as the communication regime. In this paper, without loss of the generality, we adopt the original HK model as the foundation model. We could propose a similar study that involved the adoption of the original DW model.

The HK model [4] is briefly introduced as follows:

Consider a set of agents, $A = \{A_1, A_2, \ldots, A_N\}$ and a discrete time t, $t = 0, 1, 2, \ldots$. The crisp opinion of an agent $A_i \in A$ at time t is represented by $x_i(t) \in [0, 1]$. Let $X(t) = (x_1(t), x_2(t), \ldots, x_N(t))^T$, be the vector of the opinions of all of the agents at time t that is called the opinion profile. Let ε_i be the bounded confidence of agent A_i. Agent A_i only considers the opinions that differ from his/her own opinion by not more than ε_i. When $\varepsilon_i = \varepsilon_j$ for $i, j = 1, 2, \ldots, N$, the model is termed the HK model with the homogeneous bounded confidence; otherwise, the model is termed the HK model with the heterogeneous bounded confidence.

The process of the HK model includes three steps:

(1) Determinations of the confidence sets.
Let $I(A_i, x_i(t))$ be the confidence set of agent A_i at time t, and $I(A_i, x_i(t))$ is determined as:

$$I(A_i, x_i(t)) = \{A_j \,|\, |x_i(t) - x_j(t)| \le \varepsilon_i, j = 1, 2, \ldots, N\},\quad i = 1, 2, \ldots, N. \tag{1}$$

(2) Calculations of the weights.
Let $w_{ij}(t)$ be the weight that agent A_i assigns to agent A_j at time t. Using the obtained set $I(A_i, x_i(t))$, we can calculate the weight $w_{ij}(t)$ as:

$$w_{ij}(t) = \begin{cases} 0, & A_j \notin I(A_i, x_i(t)) \\ 1/|I(A_i, x_i(t))|, & A_j \in I(A_i, x_i(t)) \end{cases},\quad i = 1, 2, \ldots, N, \tag{2}$$

where $|\bullet|$ denotes the cardinality of $I(A_i, x_i(t))$. Clearly, $w_{ij}(t) \ge 0$ and $\sum_{j=1}^{N} w_{ij}(t) = 1$.

(3) Evolutions of the opinions.
The evolutions of the opinions in the HK model are modelled as the weighted arithmetic means of the opinions in the confidence sets, i.e.,

$$x_i(t+1) = w_{i1}(t)x_1(t) + w_{i2}(t)x_2(t) + \cdots + w_{iN}(t)x_N(t),\quad i = 1, 2, \ldots, N. \tag{3}$$

3. The proposed model

In this section, we propose the interval opinion dynamics model with the dynamic bounded confidence. In our proposal, agent $A_i \in A$ expresses his/her opinion by a numerical interval $x_i(t) = [x_i^L(t), x_i^U(t)]$, where $x_i(t) \subseteq [0, 1]$. Let $X^L(t) = (x_1^L(t), x_2^L(t), \ldots, x_N^L(t))^T$ and $X^U(t) = (x_1^U(t), x_2^U(t), \ldots, x_N^U(t))^T$ be the lower and upper bounds of the opinion profile, respectively. Let $\varepsilon_i(t)$ be the dynamic bounded confidence of agent A_i at time t, where $0 \le \varepsilon_i(t) \le \alpha$, and α is the maximum threshold of the dynamic bounded confidence.

Inspired by the HK model, we define the new confidence set $\tilde{I}(A_i, x_i(t))$ as:

$$\tilde{I}(A_i, x_i(t)) = \{A_j | d(x_i(t), x_j(t)) \le \varepsilon_i(t), j = 1, 2, \ldots, N\},\quad i = 1, 2, \ldots, N, \tag{4}$$

where $d(x_i(t), x_j(t))$ denotes the distance between the interval opinions $x_i(t)$ and $x_j(t)$.

The distance $d(x_i(t), x_j(t))$ can be calculated using various distance measures (e.g., Manhattan or Euclidean).

When adopting the Manhattan distance measure, the distance $d(x_i(t), x_j(t))$ is given by

$$d(x_i(t), x_j(t)) = \frac{\left|x_i^L(t) - x_j^L(t)\right| + \left|x_i^U(t) - x_j^U(t)\right|}{2}. \tag{5}$$

When adopting the Euclidean distance measure, the distance $d(x_i(t), x_j(t))$ is given by

$$d(x_i(t), x_j(t)) = \sqrt{\frac{1}{2}\left[\left(x_i^L(t) - x_j^L(t)\right)^2 + \left(x_i^U(t) - x_j^U(t)\right)^2\right]}. \tag{6}$$

Regardless of which of the above distance measures we use, similar results are obtained. Without loss of generality, we adopt the Euclidean distance measure in the present study.

The evolutions of the interval opinions in the proposed model are modelled as the weighted arithmetic means of the upper and lower bounds of the interval opinions in the confidence sets, i.e.,

$$x_i^L(t+1) = w_{i1}(t)x_1^L(t) + w_{i2}(t)x_2^L(t) + \cdots + w_{iN}(t)x_N^L(t),\quad i = 1, 2, \ldots, N, \tag{7}$$

and

$$x_i^U(t+1) = w_{i1}(t)x_1^U(t) + w_{i2}(t)x_2^U(t) + \cdots + w_{iN}(t)x_N^U(t),\quad i = 1, 2, \ldots, N, \tag{8}$$

respectively, where the weight $w_{ij}(t)$ is given by

$$w_{ij}(t) = \begin{cases} 0, & A_j \notin \tilde{I}(A_i, x_i(t)) \\ 1/|\tilde{I}(A_i, x_i(t))|, & A_j \in \tilde{I}(A_i, x_i(t)) \end{cases},\quad i = 1, 2, \ldots, N. \tag{9}$$

Let $W(t) = (w_{ij}(t))_{N \times N}$. Then, based on Eqs. (5) and (6), the lower and upper bounds of the opinion profile are given by

$$x^L(t+1) = W(t)x^L(t). \tag{10}$$

$$x^U(t+1) = W(t)x^U(t). \tag{11}$$

The opinion profiles are further determined as

$$x^L(t+1) = W(t)W(t-1)\cdots W(0)x^L(0). \tag{12}$$

$$x^U(t+1) = W(t)W(t-1)\cdots W(0)x^U(0). \tag{13}$$

The proposed model shares a strong linkage to the HK model. If $\varepsilon_i(t) = \varepsilon_i(t+1)$ and $\varepsilon_i(t) = \varepsilon_j(t)$ are satisfied for $i, j = 1, 2, \ldots, N$ and $t = 0, 1, \ldots$, then the proposed model is reduced to the HK model with the homogeneous bounded confidence. If only $\varepsilon_i(t) = \varepsilon_i(t+1)$ is satisfied for $i = 1, 2, \ldots, N$ and $t = 0, 1, \ldots$, then the proposed model is reduced to the HK model with the heterogeneous bounded confidence.