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Interval-valued intuitionistic fuzzy mathematical programming method for hybrid multi-criteria group decision making with interval-valued intuitionistic fuzzy truth degrees

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ABSTRACT

As an important component of group decision making, the hybrid multi-criteria group decision making (MCGDM) is very complex and interesting in real applications. The purpose of this paper is to develop a novel interval-valued intuitionistic fuzzy (IVIF) mathematical programming method for hybrid MCGDM considering alternative comparisons with hesitancy degrees. The subjective preference relations between alternatives given by each decision maker (DM) are formulated as an IVIF set (IVIFS). The IVIFSs, intuitionistic fuzzy sets (IFSs), trapezoidal fuzzy numbers (TrFNs), linguistic variables, intervals and real numbers are used to represent the multiple types of criteria values. The information of criteria weights is incomplete. The IVIFS-type consistency and inconsistency indices are defined through considering the fuzzy positive and negative ideal solutions simultaneously. To determine the criteria weights, we construct a novel bi-objective IVIF mathematical programming of minimizing the inconsistency index and meanwhile maximizing the consistency index, which is solved by the technically developed linear goal programming approach. The individual ranking order of alternatives furnished by each DM is subsequently obtained according to the comprehensive relative closeness degrees of alternatives to the fuzzy positive ideal solution. The collective ranking order of alternatives is derived through establishing a new multi-objective assignment model. A real example of critical infrastructure evaluation is provided to demonstrate the applicability and effectiveness of this method.

1. Introduction

With the drastic development of modern information technology and economy, many decision making problems become more and more complex. It is very difficult or unrealistic for a single decision maker (DM) or expert to cope with a complex or important issue. Thus, group decision making (GDM) has attracted intensive concern in the decision analysis area. As an important component of GDM, fuzzy multi-criteria group decision making (MCGDM) (or multi-attribute group decision making (MAGDM)) is an intractable research subject due to the fuzziness and uncertainty of objective things and human thinking [1].

As the generalizations of fuzzy sets [2], the intuitionistic fuzzy (IF) set (IFS) [3] and interval-valued intuitionistic fuzzy (IVIF) set (IVIFS) [4] are the powerful tools to deal with imperfect and imprecise information. A prominent characteristic of IFS and IVIFS is that they assign to each element a membership degree and a non-membership degree. Over the last decades, many researchers have investigated the theories of IFS and IVIFS and applied to various fields, such as decision making [5–8], image fusion [9], and so on. Considering the prioritization relationship over attributes, Yu and Xu [5] defined the prioritized intuitionistic fuzzy aggregation operators. Li and He [6] investigated the intuitionistic fuzzy PRI-AND and PRI-OR aggregation operators and applied to multi-attribute decision making (MADM) problem under intuitionistic fuzzy environment. Zhang [7] claimed that these prioritized aggregation operators ignored the relationship between the values being fused. Thus, he developed a series of generalized intuitionistic fuzzy power geometric operators and employed these aggregation operators to propose some methods for MAGDM with IFSs. Liu

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et al. [8] constructed a partial binary tree DEA-DA cyclic classification model for decision makers in complex multi-attribute large-group interval-valued intuitionistic fuzzy decision-making problems.

Generally, MCGDM involves multiple different qualitative and quantitative criteria, the assessments of these criteria may be expressed with different formats, such as real numbers, intervals, linguistic variables, trapezoidal fuzzy numbers (TrFNs), IFSs, IVIFSs. Such a MCGDM with multiple formats of information is called the hybrid (or heterogeneous) MCGDM, which is very complex and interesting in real applications [10–15]. The aforementioned recent and interesting works [4–8] seem to be efficient to handle MADM or MCGDM with IFSs or IVIFSs. However, they only considered the assessment information of criteria as single types (i.e., IFSs or IVIFSs) and cannot be used to solve hybrid MCGDM problems. Therefore, incorporating IFSs and IVIFSs into the hybrid MCGDM is of great importance for scientific research and actual application.

Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP) developed by Srinivasan and Shocker [16] is one of typical methods for solving MCGDM problems. It is based on pair-wise comparisons of alternatives given by the DM and generates the best compromise alternative as the solution that has the shortest distance to the ideal solution (IS). The main advantages of LINMAP lie in two aspects: (1) It not only considers the preferences of DM on pair-wise comparisons of alternatives but also takes the assessments of alternatives on multiple attributes into account; (2) Through constructing linear programming of minimizing the consistency index, LINMAP can derive objectively the attribute weights and IS.

Currently, the classic LINMAP [16] has been extended and some fuzzy LINMAP methods [17–21] have been proposed under fuzzy or IF or IVIF environments. Nevertheless, there are some limitations in these existing LINMAP methods [16–21]. First, in the classic LINMAP [16], all decision data are known precisely or given as crisp values. Owing to inherent complexity and uncertainty in real-life decision problems, it is often impractical to require a DM to provide his/her judgment in precise numerical values. The fuzzy sets [2], linguistic variables [22], TrFNs [23], IFSs [3], IVIFSs [4] are usually more adequate or sufficient to model real-life decision problems than real numbers. Second, the classic LINMAP [16] and fuzzy LINMAP methods [17–21] also only considered single type of attributes and cannot deal with hybrid MCGDM. Third, in the classical LINMAP [16] and fuzzy LINMAP [17–21], the DM gives pair-wise comparisons of alternatives in the form of the ordered pairs with crisp truth degrees 0 or 1. However, in the real world, DM is not sure enough in all comparisons and may express his/her opinion with a fuzzy truth degree [24–29]. Although Sadi-Nezhad and Akhtari [24] considered the fuzzy truth degree as a triangular fuzzy number (TFN) and proposed the possibility LINMAP in MAGDM, it still only considered single type of attributes and was not suitable for hybrid MCGDM. Li and Wan [25] represented the fuzzy truth degree as a TrFNs and proposed the fuzzy linear programming method of multi-attribute decision making (MADM). The method [25] is only an extension of possibility LINMAP [24]. Li and Wan [26] developed fuzzy heterogeneous MADM method for outsourcing provider selection integrating technique for order preference by similarity to ideal solution (TOPSIS) [30] and LINMAP. Due to time pressure, lack of knowledge (or data), and limited expertise about the problem domain, DMs usually give the pair-wise comparisons of alternatives with some hesitancy degrees [27,28]. Thus, Wan and Li [27,28] used IFSs to represent IF truth degrees of alternatives' comparisons and proposed fuzzy heterogeneous MADM and MAGDM methods. Zhang and Xu [29] developed interval programming method for hesitant fuzzy MAGDM with interval truth degrees on alternatives' comparisons.

It is not difficult to find that the LINMAP has been extended into various forms for solving MADM or MCGDM problems under a variety of different environments. However, these LINMAP-based methods are not appropriate for the MCGDM problems with IVIF truth degrees on alternatives' comparisons. In fact, since IVIFSs use the intervals to characterize the membership and non-membership degrees, while IFSs use the real numbers to represent the membership and non-membership degrees, IVIFSs have stronger ability and flexibility to express the hesitant fuzzy truth degrees than IFSs. Therefore, it is necessary and natural to extend the LINMAP to suit hybrid MCGDM problems considering comparisons of alternatives with IVIF truth degrees. In this paper, we firstly use the IVIFSs to capture the hesitant fuzzy truth degrees of alternative comparisons and develop a new IVIF mathematical programming method for solving hybrid MCGDM with IVIF truth degrees and incomplete weight information. The main motivations of the proposed method in this paper are summarized as follows:

(1) The existing LINMAP [16–21,24–29] only minimized the inconsistency index, which cannot assure that the consistency index achieves the maximum. In fact, the inconsistency and consistency indexes are of equal importance in GDM. Minimizing the inconsistency index and meanwhile maximizing consistency index should be taken into account, which is the biggest motivation of this paper.
(2) The methods [24–29] considered the fuzzy truth degrees of alternatives' comparisons, they respectively utilized TFNs, TrFNs, IFSs, and intervals to characterize the fuzzy truth degrees. Nevertheless, IVIFSs have stronger ability and flexibility to express the uncertainty and hesitancy than IFSs since IVIFSs represents the membership and non-membership degrees by two closed intervals of the interval $[0,1]$. In the real-life hybrid MCGDM, DMs are not always certain about their given preference information and they often have some degree of uncertainty. It is more suitable to utilize IVIFSs to capture the fuzzy truth degrees, which is the second motivation of this paper.
(3) Most of the prior studies [16–21,24,25,17–29] only adopted the distances of alternatives to fuzzy positive ideal solution (FPIS) to rank alternatives and ignored the fuzzy negative ideal solution (FNIS). According to TOPSIS [30], the FNIS is as important as the FPIS during the process of decision making. To remedy this flaw of ignoring the FNIS, we define the IVIFS-type consistency and inconsistency by the relative closeness degree on the basis of TOPSIS, which can make decision results more reasonable and convincing.
(4) The methods [17,19–21,24,28,29] utilized the social choice functions (such as Borda's score and Copeland's function) to obtain the group ranking order of alternatives. It could happen that there exist two or more alternatives with the same Borda's scores, i.e., a total order of the set of alternatives are not guaranteed. This fact leads us to construct a new multi-objective assignment model to derive the collective ranking order of alternatives. This model ensures that different alternatives are ranked in different positions and can effectively overcome this limitation.

The rest of this paper is planned as follows. Section 2 reviews some concepts related to IVIFSs, IFSs, TrFNs as well as linguistic variables. In Section 3, the hybrid MCGDM problems with IVIF truth degrees and incomplete weight information are described. The normalization method is also presented. In Section 4, a new bi-objective IVIF mathematical programming model is constructed and solved by the developed linear goal programming approach. Hereby, the GDM method is then proposed for solving this sort of hybrid MCGDM problems. Section 5 illustrates the proposed