Information Fusion 26 (2015) 122–135

Contents lists available at ScienceDirect

Information Fusion

journal homepage: www.elsevier.com/locate/inffus

Two new methods for deriving the priority vector from interval multiplicative preference relations

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A R T I C L E I N F O

Article history:
Received 25 June 2014
Received in revised form 13 November 2014
Accepted 13 December 2014
Available online 23 December 2014

Keywords:
Decision analysis
AHP
Eigenvalue method
Row geometric mean method
Interval multiplicative preference relation

A B S T R A C T

Interval preference relations are widely used in the analytic hierarchy process (AHP) for their ability to express the expert's uncertainty. The most crucial issue arises when deriving the interval priority vector from the interval preference relations. Based on two of the most commonly used prioritization methods (the eigenvalue method (EM) and the row geometric mean method (RGMM)), two new methods for obtaining the interval priority vector from interval multiplicative preference relations are developed, which endow the expert with different risk preferences for his/her interval judgments. In contrast to existing methods, new approaches calculate the interval priority weights of alternatives separately. Then, several concepts of acceptable consistency for interval multiplicative preference relations are defined. Using a convex combination method, the acceptable consistency of interval multiplicative preference relations can be derived from the associated and exact numerical relations. To increase the distinction of intervals, an improved interval ranking method is presented. After that, two algorithms that can cope with acceptably and unacceptably consistent cases are introduced. Meanwhile, three numerical examples are examined to show the application of the new approaches, and comparisons with several other methods are also made.

1. Introduction

The analytic hierarchy process (AHP) introduced by Saaty [1] is an important tool for addressing multi-criteria decision making (MCDM) [2], which requires the expert to construct preference relations. Generally, there are two types of preference relations in the classical AHP: multiplicative preference relations [1,3] and linguistic preference relations [4–8]. In the process of group decision making using AHP, it usually includes two main procedures: (i) consistency and consensus analyses [9–17] and (ii) solving the priority vector [18–24].

Although there are many advantages of the classical AHP, it requires the expert to offer preference relations with exact values that limit its application. With socioeconomic development, the decision-making problems become more and more complex. In some situations, it is difficult or even impossible for the expert to give exact judgments. Based on fuzzy set theory [25], some researchers have paid attention to preference relations with fuzzy information, such as interval preference relations [26,27], triangular fuzzy preference relations [28], and trapezoidal fuzzy preference relations [29]. Because preference relations with fuzzy information can clearly address the situation where the expert cannot estimate his/her preference relations with exact values, these preference relations have become an important research topic.

Because interval preference relations can clearly express the lower and upper limits of the expert's uncertainty, they have been widely used in decision making. Saaty and Vargas [26] first researched the AHP with interval multiplicative preference relations and developed a Monte Carlo simulation approach for obtaining the interval priority vector. Since then, Arbel [30] constructed a linear programming model to obtain the interval priority vector from interval multiplicative preference relations. Arbel and Vargas [31] treated interval bounds as hard constraints and explored two approaches for generating the priority vector when preferences are expressed as intervals: a simulation method and a mathematical programming method. Following the work of Arbel and Vargas [32] and the logarithmic least squares method (LLSM) [9], Chandran et al. [33] presented an approach using the linear

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http://dx.doi.org/10.1016/j.inffus.2014.12.002
1566-2535/

programming models to estimate the interval priority vector from interval multiplicative preference relations, which was also utilized by Wang et al. [34]. Because this method is divided into two stages, it is also called the two-stage logarithmic goal programming method. Furthermore, Xia and Xu [35] considered interval bounds as constraints and developed a goal programming model for deriving the interval priority vector. Islam et al. [36] developed a lexicographic goal programming (LGP) method to determine weights from inconsistent interval multiplicative preference relations and explored several properties and advantages of the LGP method. However, this method is defective in theory [34]. Later, Podinovski [37] introduced a symmetrical- lexicographic goal programming (SLGP) method to obtain weights from both consistent and inconsistent interval multiplicative preference relations. Haines [38], Moreno-Jiménez [39] or Zhu and Xu [40] introduced several interesting approaches for evaluating the interval priority vector from interval multiplicative preference relations using stochastic preference analysis. Utilizing a convex combination method, Liu [41] or Liu and Lan [42] studied the acceptable consistency of interval multiplicative preference relations, considering two crisp multiplicative preference relations. It is worth noting that this acceptable consistency only holds for the expert having the same risk preference for all of his/her interval judgments. Following the works of Liu [41] and Liu and Lan [42], Liu et al. [43] researched the interval priority vector from incomplete interval multiplicative preference relations using the building model, and Liu et al. [44] developed a group decision-making model for interval additive preference relations by considering the associated interval multiplicative preference relation. Very recently, Dong and Herrera-Viedma [45] introduced a novel and interesting method to address group decision making with linguistic information. In this paper, the authors built a linear programming model to obtain interval numerical indexes of linguistic terms, which is based on a 2-tuple linguistic model [5]. Then, the authors used these interval numerical indexes to construct interval multiplicative preference relations and then calculated the interval priority vector. This gives us a new way to address linguistic preference relations. From the relationship between multiplicative and fuzzy preference relations [46], one can easily find that any interval fuzzy preference relation can be transformed into an interval multiplicative preference relation. Based on this fact, this paper focuses on studying interval multiplicative preference relations.

At present, studies about interval multiplicative preference relations can be classified into two families. One family based on the consistency concept [32,41–43,47–49], and the other family uses the associated programming model [34,35,50,51]. The main disadvantage of the latter is its failure to handle the inconsistent case. In [32,48,49], the authors considered an interval multiplicative preference relation to be consistent if it contains a consistent multiplicative preference relation. As shown in the given examples, the feasible region of the programming models in [32,49] may be empty for the inconsistent case, while Wang et al. [48] derived the interval priority vector using an eigenvector method-based nonlinear programming model that is not based on an consistent interval multiplicative preference relation [32]. To judge the consistence of interval multiplicative preference relations and to address the inconsistent case, in [41–43,47], the authors adopted a convex combination method to define a consistency concept of interval multiplicative preference relations, which is based on the assumption that the expert has the same risk preference for all of his/her interval judgments. Furthermore, all of the above methods for calculating the interval priority vector are based on the same constraints. As we know, interval preference relations indicate the experts' uncertainty for his/her judgments, whether their uncertainty is the same for all interval judgments, and whether their risk preferences are the same for all interval judgments. To cope with these issues, this paper develops two new approaches to derive the interval priority vector from interval multiplicative preference relations. Consider that the experts' risk preferences may be different for his/her interval judgments. The new methods calculate the interval priority weights separately.

Section 2 briefly reviews several (acceptable) consistency concepts of the interval multiplicative preference relations and analyses several existing issues. According to the eigenvalue method (EM) and the row geometric mean method (RGMM), Section 3 presents two approaches to derive the interval priority vector from interval multiplicative preference relations. To determine the ranges of interval weights, the associated programming model is constructed by which the upper and lower bounds of interval weights can be determined. Using a convex combination method, Section 4 studies the consistent relationship between interval multiplicative preference relations and the associated crisp relations. Meanwhile, several concepts of acceptable consistency are defined. Then, an improved interval ranking method is presented. Moreover, two algorithms are developed to cope with acceptably and unacceptably consistent cases. Section 5 presents three numerical examples to show the application of the new approaches. Meanwhile, comparisons with the other methods are given. Concluding remarks and future studies are included in Section 6.

2. Some concepts

For simplicity, let $X = \{x_1, x_2, \ldots, x_n\}$ denote the set of alternatives (or projects, criteria, and experts). To express the experts' uncertain preference relations, Saaty and Vargas [26] introduced the following interval multiplicative preference relation.

Definition 2.1 [26]. An interval multiplicative preference relation, B, is defined by

$$B = (b_{ij})_{n\times n} = \begin{pmatrix} [1,1] & [b_{12}^-, b_{12}^+] & \ldots & [b_{1n}^-, b_{1n}^+] \\ [b_{21}^-, b_{21}^+] & [1,1] & \ldots & [b_{2n}^-, b_{2n}^+] \\ \vdots & \vdots & \vdots & \vdots \\ [b_{n1}^-, b_{n1}^+] & [b_{n2}^-, b_{n2}^+] & \ldots & [1,1] \end{pmatrix},$$

where $b_{ij}^-, b_{ij}^+ \geqslant 0$ such that $b_{ij}^- \leqslant b_{ij}^+, b_{ij}^- = 1/b_{ji}^+$ and $b_{ij}^+ = 1/b_{ji}^-, b_{ij}$ indicates that x_i is between b_{ij}^- and b_{ij}^+ times as important as x_j.

When $b_{ij}^- = b_{ij}^+$ for all $i,j = 1,2,\ldots,n, B$ degenerates to a multiplicative preference relation [1]. Let $B = (b_{ij})_{n\times n}$ be an interval multiplicative preference relation, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ be the associated interval priority vector with ω_i as the interval weight of the alternative $x_i, i = 1,2,\ldots,n$. Wei et al. [52] noticed that if b_{ij} objectively reflects the ratio between ω_i and ω_j, it should have $b_{ij} = \omega_i/\omega_j$. In a similar way to Saaty [1] and Laarhoven and Pedrycz [28], Wei et al. [52] introduced the following consistency concept.

Definition 2.2 [52]. Let $B = (b_{ij})_{n\times n}$ be an interval multiplicative preference relation. If it satisfies, for all $i,j = 1,2,\ldots,n$,

$$b_{ij} = b_{ik}b_{kj} \quad \forall k = 1,2,\ldots,n,$$

then B is said to be a consistent interval multiplicative preference relation.

From Minkowski operations on intervals, one can easily see that Definition 2.2 is not true, namely, $b_{ij} = b_{ik}b_{kj}$ does not hold for all $i,j,k = 1,2,\ldots,n$. Later, following the work of Arbel and Vargas [31], Wang et al. [34,48] considered the interval bounds as constraints of the exact weight vector and introduced the following consistency concept of interval multiplicative preference relations.