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A comment on "Incomplete fuzzy linguistic preference relations under uncertain environments"

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ABSTRACT

In this comment, we first propose some new results on triangular fuzzy numbers. The results show that some definitions and the proof process of propositions proposed by Wang and Chen (2010, 2008) [2,1] do not hold in general cases. We then present reasonable definitions and re-prove the propositions.

1. Introduction

In [1,2], Wang and Chen applied the fuzzy linguistic preference relation method to enhance the consistency of fuzzy AHP method. They proposed some propositions to establish fuzzy preference relations matrices based on consistent fuzzy preference relations and fuzzy linguistic assessment variables. In this comment, we first propose some new results based on the operational laws of triangular fuzzy numbers. These results show that some definitions and the proof processes of propositions proposed by Wang and Chen do not hold in general cases. Then, we give reasonable definitions and re-prove the propositions.

2. Preliminaries

Definition 1 [3]. Assume the triangular fuzzy number (TFN) $\widetilde{A}$ on R is a fuzzy set with membership function as follows:

$$\mu_{\widetilde{A}}(x) = \begin{cases} (x-l)/(m-l), & l \leqslant x \leqslant m \\ (u-x)/(u-m), & m \leqslant x \leqslant u \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

with $l \leqslant m \leqslant u$. The TFN is denoted by $\widetilde{A} = (l, m, u)$.

Let $\widetilde{A} = (l, m, u)$, $\widetilde{A}_1 = (l_1, m_1, u_1)$ and $\widetilde{A}_2 = (l_2, m_2, u_2)$ be three positive TFNs. Then the operations on the TFNs are defined as [3,4]:

$$\text{Addition} \quad \oplus: \ \widetilde{A}_1 \oplus \widetilde{A}_2 = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2); \tag{2}$$

$$\text{Subtraction} \quad \ominus: \ \widetilde{A}_1 \ominus \widetilde{A}_2 = (l_1, m_1, u_1) \ominus (l_2, m_2, u_2) = (l_1 - u_2, m_1 - m_2, u_1 - l_2); \tag{3}$$

$$\text{Multiplication} \quad \otimes: \ \widetilde{A}_1 \otimes \widetilde{A}_2 = (l_1, m_1, u_1) \otimes (l_2, m_2, u_2) = (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2); \tag{4}$$

$$\text{Division} \quad ø: \ \widetilde{A}_1 ø \widetilde{A}_2 = (l_1, m_1, u_1) ø (l_2, m_2, u_2) = (l_1/u_2, m_1/m_2, u_1/l_2); \tag{5}$$

$$\text{Logarithm}: \ \log_n(\widetilde{A}) = (\log_n l, \log_n m, \log_n u), \ n \text{ is base.} \tag{6}$$

$$\text{Reciprocal}: \ (\widetilde{A})^{-1} = (l, m, u)^{-1} = (1/u, 1/m, 1/l). \tag{7}$$

Definition 2. Let $\widetilde{A}_1 = (l_1, m_1, u_1)$ and $\widetilde{A}_2 = (l_2, m_2, u_2)$ be two TFNs, if $\widetilde{A}_1 = \widetilde{A}_2$, then $l_1 = l_2$, $m_1 = m_2$, $u_1 = u_2$.

For a TFN $\widetilde{A} = (l, m, u)$, if $l = m = u = q$, then the TFN $\widetilde{A}$ is reduced to a crisp number, we denote $q = (q, q, q)$, especially, $0 = (0, 0, 0)$, $1 = (1, 1, 1)$.

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3. Some new results

Based on the operational laws of triangular fuzzy numbers, we can easily get the following results:

Proposition 1. *Let $\widetilde{A}_1 = (l_1, m_1, u_1)$, $\widetilde{A}_2 = (l_2, m_2, u_2)$ and $\widetilde{A}_3 = (l_3, m_3, u_3)$ be three TFNs, q = (q, q, q) is a constant crisp number, then*

(a) $\widetilde{A}_1 \oplus \widetilde{A}_2 = \widetilde{A}_3 \not\Leftrightarrow \widetilde{A}_1 = \widetilde{A}_3 \ominus \widetilde{A}_2$;

(b) $\widetilde{A}_1 \oplus \widetilde{A}_2 = \widetilde{A}_3 \Rightarrow \widetilde{A}_1 \oplus \widetilde{A}_2 \ominus \widetilde{A}_3 \neq 0$;

(c) $\widetilde{A}_1 \ominus \widetilde{A}_2 \neq q$. Especially, $\widetilde{A}_1 \ominus \widetilde{A}_1 \neq 0$;

(d) $\widetilde{A}_1 \oplus \widetilde{A}_2 \neq q$. Especially, $\widetilde{A}_1 \oplus \widetilde{A}_2 \neq 0$, $\widetilde{A}_1 \oplus \widetilde{A}_2 \neq 1$;

(e) $\log_n(\widetilde{A}_1) \oplus \log_n(\widetilde{A}_2) = \log_n(\widetilde{A}_3) \not\Leftrightarrow \log_n(\widetilde{A}_1) = \log_n(\widetilde{A}_3) \ominus \log_n(\widetilde{A}_2)$;

(f) $\log_n(\widetilde{A}_1) \oplus \log_n(\widetilde{A}_2) = \log_n(\widetilde{A}_3) \Rightarrow \log_n(\widetilde{A}_1) \oplus \log_n(\widetilde{A}_2) \ominus \log_n(\widetilde{A}_3) \neq 0$;

(g) $\log_n(\widetilde{A}_1) \oplus \log_n(\widetilde{A}_2) \neq q$. Especially, $\log_n(\widetilde{A}_1) \oplus \log_n(\widetilde{A}_2) \neq 0$;

(h) $\log_n(\widetilde{A}_1) \ominus \log_n(\widetilde{A}_2) \neq q$. Especially, $\log_n(\widetilde{A}_1) \ominus \log_n(\widetilde{A}_2) \neq 0$

Proof. (a) From the operational laws Eqs. (2)–(7), if $\widetilde{A}_1 \oplus \widetilde{A}_2 = \widetilde{A}_3$, that is

$$\widetilde{A}_1 \oplus \widetilde{A}_2 = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) = \widetilde{A}_3 = (l_3, m_3, u_3)$$

thus, by Definition 2, we have

$$l_1 + l_2 = l_3, \quad m_1 + m_2 = m_3, \quad u_1 + u_2 = u_3, \tag{8}$$

If $\widetilde{A}_1 = \widetilde{A}_3 \ominus \widetilde{A}_2$, then

$$\widetilde{A}_1 = \widetilde{A}_3 \ominus \widetilde{A}_2 = (l_3, m_3, u_3) \ominus (l_2, m_2, u_2) = (l_3 - u_2, m_3 - m_2, u_3 - l_2) = \widetilde{A}_1 = (l_1, m_1, u_1)$$

thus, by Definition 2, we have

$$l_3 - u_2 = l_1, \quad m_3 - m_2 = m_1, \quad u_3 - l_2 = u_1 \tag{9}$$

From Eqs. (8) and (9), if Eqs. (8) and (9) hold, we have $l_2 = m_2 = u_2$, that is $\widetilde{A}_2$ is a crisp number, for a general TFN $\widetilde{A}_2$, obviously, we have

$$\widetilde{A}_1 \oplus \widetilde{A}_2 = \widetilde{A}_3 \not\Leftrightarrow \widetilde{A}_1 = \widetilde{A}_3 \ominus \widetilde{A}_2$$

(b) If $\widetilde{A}_1 \oplus \widetilde{A}_2 = \widetilde{A}_3$, then

$$\widetilde{A}_1 \oplus \widetilde{A}_2 \ominus \widetilde{A}_3 = \widetilde{A}_3 \ominus \widetilde{A}_3 = (l_3 - u_3, m_3 - m_3, u_3 - l_3) \neq 0$$

Especially,

$$\widetilde{A}_1 \oplus 0 = \widetilde{A}_1,$$

but
$\widetilde{A}_1 \ominus \widetilde{A}_1 = (l_1, m_1, u_1) \ominus (l_1, m_1, u_1) = (l_1 - u_1, m_1 - m_1, u_1 - l_1) \neq 0$.
(c) By Eq. (3), we have

$$\widetilde{A}_1 \ominus \widetilde{A}_2 = (l_1, m_1, u_1) \ominus (l_2, m_2, u_2) = (l_1 - u_2, m_1 - m_2, u_1 - l_2)$$

If $\widetilde{A}_1 \ominus \widetilde{A}_2 = q = (q, q, q)$, then

$$l_1 - u_2 = q, \quad m_1 - m_2 = q, \quad u_1 - l_2 = q,$$

Thus,

$$l_1 = u_2 + q, \quad m_1 = m_2 + q, \quad u_1 = l_2 + q, \tag{10}$$

But by the Definition 1 of TFN, we know that

$$l_1 \leqslant m_1 \leqslant u_1, \quad l_2 \leqslant m_2 \leqslant u_2.$$

Therefore, Eq. (10) holds, if and only if

$$l_2 = m_2 = u_2, \quad l_1 = m_1 = u_1 = l_2 + q = m_2 + q = u_2 + q,$$

which means $\widetilde{A}_1$ and $\widetilde{A}_2$ are crisp numbers. Thus, for the general TFN $\widetilde{A}_1$ and $\widetilde{A}_2$, we have

$$\widetilde{A}_1 \ominus \widetilde{A}_2 \neq q,$$

Especially, $\widetilde{A}_1 \ominus \widetilde{A}_1 \neq 0$.
(d) By Eq. (2), we have

$$\widetilde{A}_1 \oplus \widetilde{A}_2 = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$$

If $\widetilde{A}_1 \oplus \widetilde{A}_2 = q = (q, q, q)$, then

$$l_1 + l_2 = q, \quad m_1 + m_2 = q, \quad u_1 + u_2 = q, \tag{11}$$

But by the Definition 1 of TFN, we know that

$$l_1 \leqslant m_1 \leqslant u_1, \quad l_2 \leqslant m_2 \leqslant u_2$$

Therefore, Eq. (11) holds if and only if

$$l_1 = m_1 = u_1, \quad l_2 = m_2 = u_2 = q - l_1 = q - m_1 = q - u_1,$$

which means $\widetilde{A}_1$ and $\widetilde{A}_2$ are crisp numbers. Thus, for the general TFNs $\widetilde{A}_1$ and $\widetilde{A}_2$, we have

$$\widetilde{A}_1 \oplus \widetilde{A}_2 \neq q$$

Especially, $\widetilde{A}_1 \oplus \widetilde{A}_2 \neq 0$, $\widetilde{A}_1 \oplus \widetilde{A}_2 \neq 1$.

Therefore, (a)–(d) are proved.

Similarly, based on the operational law (Eq. (6)) of fuzzy number logarithm, (e)–(h) can be proved easily. The Proposition 1 shows addition and subtraction, multiplication and division operational laws of TFNs, are not mutually inverse. It is different from the operation laws of crisp numbers. □

4. The new definitions and re-proof of the propositions proposed by Wang and Chen

Saaty [5] first introduced the reciprocal preference relation in the AHP. The concept is as follows.

A reciprocal preference relation A on a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ is represented by a matrix $A \subset X \times X$, $A = (a_{ij})_{n\times n}$, where a_{ij} denotes the preference intensity of the alternatives x_i over x_j on a 1–9 scale, where $1/9 \leqslant a_{ij} \leqslant 9$, $a_{ii} = 1$, $a_{ij} \cdot a_{ji} = 1$ for all $i, j = 1, 2, \ldots, n$. Moreover, the definition of consistent reciprocal preference relation is stated as follows:

Definition 3. A reciprocal preference relation $A = (a_{ij})_{n\times n}$ is consistent if $a_{ij} = a_{ik}a_{kj}$ for all $i, j, k = 1, 2, \ldots, n$.

Based on Definition 3, Wang and Chen [1,2] extended the above definition to the triangular fuzzy reciprocal preference relation and give the following propositions.

Definition 4 [1]. The triangular fuzzy positive matrix $\widetilde{A} = (\tilde{a}_{ij})_{n\times n}$ is reciprocal if and only if $\tilde{a}_{ij} \otimes \tilde{a}_{ji} = 1$.

By (g) of Proposition 1, we know that Definition 4 do not hold in general fuzzy cases. It should be corrected as follows:

Definition 5. A triangular fuzzy positive matrix $\widetilde{A} = (\tilde{a}_{ij})_{n\times n}$ is reciprocal if and only if $a_{ji} = a_{ij}^{-1}$.

Definition 6 ([1,2]). The triangular fuzzy positive reciprocal matrix $\widetilde{A} = (\tilde{a}_{ij})_{n\times n}$ is consistent if and only if $\tilde{a}_{ij} \otimes \tilde{a}_{jk} \approx \tilde{a}_{ik}$.

In the following, we will show that the Definition 6 is also not properly.

Proof. Let $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^R)$, $\tilde{a}_{jk} = (a_{jk}^L, a_{jk}^M, a_{jk}^R)$, $\tilde{a}_{ik} = (a_{ik}^L, a_{ik}^M, a_{ik}^R)$, if $\tilde{a}_{ij} \otimes \tilde{a}_{jk} \approx \tilde{a}_{ik}$, according to the operational of triangular fuzzy numbers Eq. (4), we have

$$a_{ij}^L \times a_{jk}^L = a_{ik}^L, \quad a_{ij}^M \times a_{jk}^M = a_{ik}^M, \quad a_{ij}^R \times a_{jk}^R = a_{ik}^R,$$