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A cardinality modified product multi-sensor PHD

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ABSTRACT

The iterated-corrector PHD (IC-PHD) filter, which is the most commonly used multi-sensor PHD filter, is affected by the sensor order and the probability of detection. To address this problem, the product multi-sensor PHD (PM-PHD) filter, a modified version of the IC-PHD filter, is proposed. The update formulation of the PM-PHD filter consists of a likelihood function and a modified coefficient. Although the coefficient improves the performance of the PM-PHD filter, it still has some drawbacks. In this paper, two improvements on the coefficient are proposed. (1) The coefficient is the quotient of two infinite sums which are computational intractable. We prove that some terms in the infinite sums can be eliminated, and thus the infinite sums can be approximated by the sum of finite terms. (2) Since the coefficient is a scalar quantity, it mainly focuses on maintaining the magnitudes of the posterior PHD and the number of targets. It may lead to an inaccurate state estimation in some situations. In the Gaussian mixture implementation of the PM-PHD filter, a cardinality modified method is proposed to reassign the weight of Gaussian components and modify the posterior PHD. The advantages of these two methods are verified by simulations and experiments.

1. Introduction

Multi-target tracking (MTT) aims to estimate the motion state of targets from the measurement information. The traditional MTT algorithms, which are mainly designed on the data association, include multiple hypotheses tracking (MHT) [1,2], joint probabilistic data association (JPDA) [3] and their variants [4–9]. With the number of targets and the clutter intensity increasing, a high computation burden caused by data association reduces the real-time performances of these above algorithms. In recent years, Mahler proposed an effective approach for the MTT problem, named the random finite set (RFS) theory [10]. The number of targets and the dimension of state space are random variables for the reason that the number of targets may vary with time. Thus, the state model and the observation model can be represented by random finite set, and many derivative algorithms [11–18] are proposed on the basis of the RFS theory. Among them, the probability hypothesis density (PHD) filter [14], the cardinalized probability hypothesis density (CPHD) filter [15,16] and the multi-target multi-Bernoulli (MeMBer) filter [10] are the most commonly used methods. Unlike the PHD filter, the CPHD filter propagates the multi-target posterior density and the posterior cardinality distribution simultaneously. Although the CPHD filter is more accurate than the PHD filter in the cardinality estimation, the CPHD filter has a higher complexity $O(m^3 \cdot n)$, where m and n are the numbers of measurements and targets. To reduce the complexity of the CPHD filter, a linear-complexity CPHD (LC-CPHD) filter [17] is proposed. Unlike the PHD filter and the CPHD filter, the MeMBer filter [10] has a cardinality bias. To deal with this problem, a cardinality-balanced MeMBer (CBMeMBer) filter [18] is proposed. Recently, Vo and Vo introduced an RFS with distinct labels, named the label RFS [19]. Furthermore, they proposed two algorithms depending on the label RFS, the generalized labeled multi-Bernoulli (GLMB) filter [20] and the labeled multi-Bernoulli (LMB) filter [21]. These two filters are the generalizations of the MeMBer filter without the cardinality bias, which can estimate the target tracks under low probability of detection and high clutter density scenarios. The above two filters have been applied in visual tracking [22], extended target tracking [23] and sensor control [24]. Although the RFS algorithms achieve well performance, they need to obey the assumptions that the birth intensity, the clutter intensity and the probability of detection are known a priori. To address this drawback, many research studies [25–27] have been carried out.

With the development of sensor technologies, considerable attention has been focused on multi-sensor solution [28] for both nonmilitary and military applications, such as robotics [29], image processing [30] and target tracking [31]. For the MTT problem, the multi-sensor solution combines the information obtained from different sensors to improve estimates of target state and velocity. Compared with single-sensor target tracking, multi-sensor target

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tracking achieves a better performance on the detection, the stability and the field of view of tracking system. In [14], Mahler proposed a simple and feasible multi-sensor algorithm, called the Iterated-corrector PHD (IC-PHD) filter. In the IC-PHD filter, the updated PHD of each sensor is the predicted PHD of the next one, and the updated PHD of the last sensor is considered as tracking results. In [32], it is pointed out that the IC-PHD filter is affected by the sensor order and sensitive to the probability of detection. The product multi-sensor PHD (PM-PHD) filter [33], an improved version of the IC-PHD filter, is proposed to deal with this problem. With an additional modified coefficient, the PM-PHD filter is more accurate than the IC-PHD filter for estimating the cardinality. However, the coefficient is the quotient of two infinite sums, and it is incapable of computing or approximating the two infinite sums. In this regard, an approximate solution for the infinite sums is proposed in this paper. Furthermore, a heuristic method is proposed to handle the problems caused by the modified coefficient. In the GM implementation of the PM-PHD filter, the heuristic method reassigns the weights of Gaussian components and modifies the posterior intensity. Different parameters, such as the probability of detection, the clutter intensity and the observation noise, are chosen to test the performance of the proposed algorithm. Simulation results show that both the cardinality estimation and the state estimation are improved by the heuristic method.

The rest of this paper is organized as follows. The RFS and the PHD filter are described in Section 2. The PM-PHD filter is introduced in Section 3. The approximate solution for the infinite sums and the cardinality modified method are presented in Sections 4 and 5, respectively. Simulation results are performed in Section 6. Finally, Section 7 gives the conclusions.

2. Background

In the multi-target motion model, the state set and the observation set can be represented by $\mathbf{X}_k = \{\mathbf{x}_1, \ldots, \mathbf{x}_{N_k}\}$ and $\mathbf{Z}_k = \{\mathbf{z}_1, \ldots, \mathbf{z}_{M_k}\}$, respectively. Here, N_k and M_k denote the numbers of targets and measurements at time k.

Given a state RFS $\mathbf{X}_{k-1}$ at time $k-1$, the state RFS $\mathbf{X}_k$ at time k can be expressed by

$$\mathbf{X}_k = \left(\bigcup_{\xi \in \mathbf{X}_{k-1}} \mathbf{S}_{k|k-1}(\xi) \right) \cup \left(\bigcup_{\xi \in \mathbf{X}_{k-1}} \mathbf{B}_{k|k-1}(\xi) \right) \cup \mathbf{\Gamma}_k \tag{1}$$

where $\mathbf{S}_{k|k-1}(\xi)$ represents the RFS of targets which still survive at time k from $\xi \in \mathbf{X}_{k-1}$. $\mathbf{B}_{k|k-1}(\xi)$ represents the RFS of targets spawned by $\xi \in \mathbf{X}_{k-1}$. $\mathbf{\Gamma}_k$ represents the RFS of new targets which appear instantly at time k.

Given a state RFS $\mathbf{X}_k$ at time k, the observation RFS $\mathbf{Z}_k$ can be expressed by

$$\mathbf{Z}_k = \mathbf{K}_k \cup \left(\bigcup_{\xi \in \mathbf{X}_k} \mathbf{\Theta}_k(\xi) \right) \tag{2}$$

where $\mathbf{K}_k$ represents the observation set of clutter, and $\mathbf{\Theta}(\boldsymbol{\xi})$ represents the observation set generated by the state $\boldsymbol{\xi}$.

Let $D_{k|k-1}(\mathbf{x})$ and $D_k(\mathbf{x})$ denote the PHDs of the predicted density $p_{k|k-1}(\mathbf{x})$ and the posterior density $p_k(\mathbf{x})$ at time k, respectively. Then the posterior intensity can be derived by the PHD recursion,

$$D_{k|k-1}(\mathbf{x}) = \int p_{S,k}(\xi) f_{k|k-1}(\mathbf{x}|\xi) D_{k-1}(\xi) d\xi + \int b_{k|k-1}(\mathbf{x}|\xi) D_{k-1}(\xi) d\xi + \gamma_k(\mathbf{x}) \tag{3}$$

$$D_{k|k}(\mathbf{x}) = \left[1 - p_{D,k}(\mathbf{x})\right] D_{k|k-1}(\mathbf{x}) + \sum_{\mathbf{z} \in \mathbf{Z}_k} \frac{p_{D,k}(\mathbf{x}) g_k(\mathbf{z}|\mathbf{x}) D_{k|k-1}(\mathbf{x})}{\kappa_k(\mathbf{z}) + \int p_{D,k}(\xi) g_k(\mathbf{z}|\xi) D_{k|k-1}(\xi) d\xi} \tag{4}$$

where $\gamma_k(\cdot)$ denotes the PHD of the birth RFS $\mathbf{\Gamma}_k$ at time k, $b_{k|k-1}(\cdot|\xi)$ denotes the PHD of the spawned RFS $\mathbf{B}_{k|k-1}(\xi)$ at time k, and $\kappa_k(\cdot)$ denotes the intensity of the clutter RFS $\mathbf{K}_k$. $p_{S,\,k}(\cdot)$ and $p_{D,\,k}(\cdot)$ are the probabilities of survival and detection, respectively. $f_{k|k-1}(\cdot|\xi)$ and $g_k(\cdot\,|\,\cdot)$ are the state transition function and the observation likelihood function, respectively.

3. Product multi-sensor PHD filter

Suppose that there are s sensors. The measurement set of the ith sensor is denoted by $\mathbf{Z}_k^i = \{\mathbf{z}_1, \ldots, \mathbf{z}_{\overset{i}{m}}\}, i = 1, \ldots, s$, where $\overset{i}{m}$ is the number of measurements.

The PM-PHD filter can be written as

$$\overset{1\ldots s}{D}_{P,k|k}(\mathbf{x}) = K_{\mathbf{Z}_k^1,\ldots,\mathbf{Z}_k^s} \cdot L^1{}_{\mathbf{Z}_k^1}(\mathbf{x}) \cdots L^s{}_{\mathbf{Z}_k^s}(\mathbf{x}) \cdot \overset{1\ldots s}{D}_{k|k-1}(\mathbf{x}) \tag{5}$$

$$L^i{}_{\mathbf{Z}_k^i}(\mathbf{x}) = 1 - p_{D,k}^i(\mathbf{x}) + \sum_{\mathbf{z} \in \mathbf{Z}_k^i} \frac{p_{D,k}^i(\mathbf{x}) L_{\mathbf{z}}(\mathbf{x})}{\kappa_k^i(\mathbf{z}) + D_{k|k-1}\left[p_{D,k}^i, L_{\mathbf{z}}\right]} \tag{6}$$

$$L_{\mathbf{z}}(\mathbf{x}) = g(\mathbf{z}|\mathbf{x}) \tag{7}$$

$$\overset{1\ldots s}{D}_{k|k-1}\left[p_{D,k}^i L_{\mathbf{z}}\right] = \int p_{D,k}^i(\xi) L_{\mathbf{z}}(\xi) D_{k|k-1}(\xi) d\xi. \tag{8}$$

Eq. (5) can be divided into two parts: the pseudo-likelihoods $L^1{}_{\mathbf{Z}_k^1}(\mathbf{x}) \cdots L^s{}_{\mathbf{Z}_k^s}(\mathbf{x})$, and the scalar quantity $K_{\mathbf{Z}_k^1,\ldots,\mathbf{Z}_k^s}$ which is used to modify the cardinality estimation. Here, $K_{\mathbf{Z}_k^1,\ldots,\mathbf{Z}_k^s} = \frac{\phi}{\upsilon_{k|k}^1 \cdots \upsilon_{k|k}^s}$, $\upsilon_{k|k}^i = \overset{1\ldots s}{s}_{k|k-1}[L^i_{\mathbf{Z}_k^i}]$, and ϕ is computed by:

$$\phi = \frac{\sum_{n \geq 0} \hat{\ell}^1_{\mathbf{Z}_k^1}(n+1) \cdots \hat{\ell}^s_{\mathbf{Z}_k^s}(n+1) \cdot e^{-\theta} \cdot \frac{\theta^n}{n!}}{\sum_{j \geq 0} \hat{\ell}^1_{\mathbf{Z}_k^1}(j) \cdots \hat{\ell}^s_{\mathbf{Z}_k^s}(j) \cdot e^{-\theta} \cdot \frac{\theta^j}{j!}} \tag{9}$$

$$\theta = \overset{1\ldots s}{N}_{k|k-1} \cdot \eta, \quad \eta = \frac{\overset{1\ldots s}{s}_{k|k-1}\left[L^1{}_{\mathbf{Z}_k^1} \cdots L^s{}_{\mathbf{Z}_k^s}\right]}{\upsilon_{k|k}^1 \cdots \upsilon_{k|k}^s} \tag{10}$$

$$\hat{\ell}^i_{\mathbf{Z}_k^i}(n) = \sum_{l=0}^{\min\left(n, \overset{i}{m}\right)} l! \cdot C_n^l \cdot \overset{1\ldots s}{s}_{k|k-1}\left[1 - p_{D,k}^i\right]^{n-l} \hat{\sigma}_l^i(\mathbf{Z}_k^i) \tag{11}$$

$$\hat{\sigma}_l^i(\mathbf{Z}_k^i) = \sigma_{\overset{i}{m},l}\left(\frac{\overset{1\ldots s}{s}_{k|k-1}\left[p_{D,k}^i L_{\mathbf{z}_1}\right]}{\kappa_k^i(\mathbf{z}_1)}, \ldots, \frac{\overset{1\ldots s}{s}_{k|k-1}\left[p_{D,k}^i L_{\mathbf{z}_{\overset{i}{m}}}\right]}{\kappa_k^i\left(\mathbf{z}_{\overset{i}{m}}\right)} \right) \tag{12}$$

$$\overset{1\ldots s}{s}_{k|k-1}[h] = \int h(\xi) \cdot \overset{1\ldots s}{s}_{k|k-1}(\xi) d\xi \tag{13}$$

$$\overset{1\ldots s}{s}_{k|k-1}(\mathbf{x}) = \frac{\overset{1\ldots s}{D}_{k|k-1}(\mathbf{x})}{\overset{1\ldots s}{N}_{k|k-1}}. \tag{14}$$

In the above equations, the superscript i denotes the sensor index; parameter with the superscript $1, \ldots, s$ indicates that this parameter is determined by all sensors.