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A group decision making model for partially ordered preference under uncertainty

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ABSTRACT

In many decision making problems, the experts are not able to provide accurate preferences among the alternatives but some kind of partial orders with certain belief degrees, due to limited expertise related to the problem domain, lack of data, or time restriction and so on. To facilitate decision making in this type of situations, this paper proposes a belief structure to represent the partially ordered preferences with belief degrees, which can cover both qualitative and quantitative aspects of the evaluation and can also represent indifference and incomparability relations as well. An evidential reasoning based preference combination approach is then applied to combine the partially ordered preferences with belief degrees of the experts. The collective ordering of alternatives, which again could be a partial order, is generated based on a distance measure between pairs of preference relations. A group decision making model based on the preference combination and the collective ordering generation is then established for partially ordered preference under uncertainty. Numerical examples are provided to illustrate the rationality and effectiveness of the proposed approach.

1. Introduction

Preference is widely involved in real world decision making problems. Examples include risk aversion in economics and finance [1], quality assessment of service, dance competition adjudication, and meta-search engine whose goal is to combine the preference relations of several WWW search engines [2], and so on. This kind of preference usually appears as a partially ordered structure [3], sometimes with certain belief degrees, due to lack of data or knowledge, time restriction, or limited expertise related to the problem domain and other factors. For example, some customer may express his/her preference over the color of the car as, "I prefer silver car to blue car with degree 0.8 (i.e., not 100% sure about this preference, but only to degree of belief), and blue to red with degree 0.9, but I felt indifferent to a black against a blue car". For decision making under this kind of situation, a preference aggregation procedure is needed to combine these partial orders with belief degrees from the different experts to produce an overall preference ordering, and this again could be a partially ordered preference.

A preference relation R is usually modelled by a preference structure, a triplet (P, I, J) of three binary relations: strict preference, indifference and incomparability [4],[1] while most of existing preference based decision making methods are based on the assumption that the experts' preferences can be linearly ordered, i.e., only strict preference relation, or sometimes with indifference relation, are considered. For example, the widely used preference relation matrix (no matter multiplicative, fuzzy, or linguistic) based on pair-wise preference is actually not able to reflect the incomparability relation between two alternatives. Actually, most preferences in reality are nonlinear, due to the fact that human decision making are usually associated with many uncertainties [5]. Incomparability is such a kind of uncertainty which appears pervasively in the real life, especially in those situations where intelligent activities of humans involve conflicting opinions or missing information. Incomparability is an important type of uncertainty, but it is not easily handled through conventional approaches due to its complexity [5–7]. For instance, we usually find it difficult to make a decision in real life when the decision is based on multiple criteria where conflicting opinions often exist. Partially ordered set is a more suitable and flexible option for information modelling and processing

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[1] In this paper, we add the inverse of the strict preference relation to the representation model, which can be easily induced from the strict preference relation.

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under these situations [8]. The motivation or the novelty of the proposed work is to provide a model with the ability to handle nonlinear ordered preference in decision making problem without oversimplified totally ordered preference assumption in most of preference involved decision making approaches.

Although some methods have been developed for handling incomplete preference relation [9–12], they usually use certain method based on additive consistency and/or the other consistency to estimate the missing values of the incomplete preference relation in order to reach a complete one before aggregation. It has actually changed the original preferences from the experts, which cannot reflect the reality accurately by doing this, although these methods utilize some consistency indices for estimating the missing values from the existing information. It should be more preferable if we can treat the preferences directly in their original forms without estimating the missing values.

For ranking alternatives based on the provided preferences in their original forms, a method has been proposed in [2,13] to calculate the probability that each pair of alternatives should be placed in an order from the preferences provided by experts, and this probability is then fed to an algorithm developed by Cohen et al. [14] to generate an approximately optimal total order for all the alternatives from pair-wise preferences. This method has provided some new ideas for calculating preference value from a different point of view by taking each preference given by experts as a sequence, but it is not expressive under situations where some alternatives can be equally ranked, i.e., are indifferent.

In order to handle partially ordered preferences for decision making, [15–18] proposed several distance-based alternative ranking approaches which take all the four preference relations (including the inverse of the strict preference relation) into consideration, but they allow only one of the four relations to be used when expressing the preferences of each expert between two alternatives, and they have not considered the degrees of credibility or belief associated with the preferences given by different experts, which are actually usually accompanying the process of experts expressing their preferences.

Belief degrees, as stated at the beginning, are often used by experts for denoting the uncertainty about their preferences. Belief function theory is one effective and formal method for modelling belief degrees, which is originally developed by Dempster [19] and Shafer [20], and so also called Dempster–Shafer (D–S) theory of evidence. It provides a framework for representing and reasoning with uncertain and incomplete information in terms of upper and lower probabilities rather than a single probability value as used in probability theory. This offers a more flexible way for constructing and analyzing "frame of discernment" and the belief allocation can vary to suit the extent of our knowledge, which is more in line with the human habit of thinking and has been applied in a wide variety of areas [21].

Based on our previous work on handling partially ordered preferences with belief degrees [22], we adopt in this paper the belief structure as a unified framework for representing the preferential opinions with uncertainty from different experts, i.e., qualitative partially ordered preference between alternatives associated with quantitative belief degrees and propose a method for transforming different types of preference relations under uncertainty into a unified form of belief structure. The evidential reasoning algorithm [23–26] based on belief function theory [19,20] is then applied to combine the preferential opinions from different experts into a collective belief structure matrix. Based on this collective belief structure matrix, two overall rankings (total orders) between the considered alternatives are elaborated by a distance-based aggregation method, and then, a partial order, the final ranking of the alternatives, is produced using the intersection of these two total orders based on ELECTRE III rules [27].

The rest of this paper is structured as follows. In Section 2, we put forward a method for transforming different types of preference relations into a unified form of belief structure. The preferences, in belief structures, from different experts are aggregated into a collective belief decision matrix based on an evidential reasoning algorithm in Section 3. Then the final ranking of the alternatives, might again be a partial order, is generated based on a distance measure and ELECTRE III interaction rules in Section 4. Numerical examples along with further discussions are provided in Section 5 to illustrate the feasibility and rationality of the proposed method. Concluding remarks are drawn in Section 6.

2. Unified representation of partially ordered preferences under uncertainty

2.1. Belief structure

The group decision making problem with partially ordered preferences under uncertainty is formalized as follows. There are a group of experts $E = \{e_1, e_2, \ldots, e_n\}$, with the corresponding weights $W = \{\omega_1, \omega_2, \ldots, \omega_n\}$ where $\omega_i \geqslant 0$, and $\sum_i \omega_i = 1$. The preferences of these experts among a set of alternatives $A = \{a_1, a_2, \ldots, a_m\}$ are not restricted to a fixed form, i.e., the preferences from different experts could be in different forms. For example, experts can express their preferences in preference relation matrix based on pair-wise comparison, sequence-like preference, e.g., $a_4a_2a_1a_5$, and $a_2a_3a_1$, or graphical form shown as Fig. 1, or the preference with belief structure [23–25] which is adopted as the uniform representation model of different forms of preference in this paper. Belief degrees can also be provided to show the credibility on different preferences under some complex and dynamic environments. The group decision making problem is then to find an appropriate approach for combining the preferences from all the experts to reach an overall rank ordering of the alternatives.

Definition 1. Suppose that $0 \leqslant \beta^t \leqslant 1$, $t = 1, 2, 3, 4$, and $\sum_{t=1}^{4} \beta^t \leqslant 1$. The belief structure of preference is defined as a 4-tuple $S(a, b) = (\beta^1, \beta^2, \beta^3, \beta^4)$, which denotes the belief degree distributions of the expert with respect to all possible four preference relations: strict preference ($\succ$), the inverse of strict preference ($\prec$ or $\succ^{-1}$), indifference ($\approx$), and incomparability ($//$) between two alternatives a and b. If $\sum_{t=1}^{4} \beta^t = 1$, we call that the belief structure of preference is complete. Otherwise, it is called incomplete.

Take the preference of the kth expert on alternative a_i and alternative a_j as an example, it is expressed as the belief distribution structure $S_k(a_i, a_j) = (\beta_k^1(i,j), \beta_k^2(i,j), \beta_k^3(i,j), \beta_k^4(i,j))$, where $\beta_k^1(i,j)$ is the belief degree with which alternative a_i is preferred to alternative a_j by the expert (denoted as $a_i \succ a_j$), $\beta_k^2(i,j)$ is the belief degree with which alternative a_j is preferred to alternative a_i (denoted as $a_i \prec a_j$), $\beta_k^3(i,j)$ is the belief degree with which alternative a_i is considered indifferent to alternative a_j (denoted as $a_i \approx a_j$), and $\beta_k^4(i,j)$ is the belief degree that the expert think alternative a_i and alternative a_j are incomparable, or the expert cannot give the preference

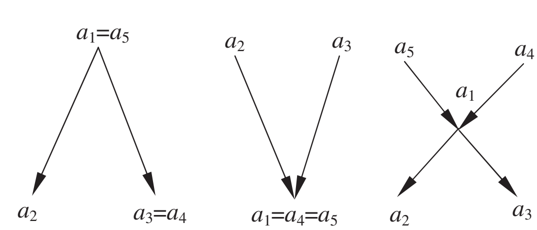

Fig. 1. Graphical forms of partially ordered preference structure.