Image and Vision Computing 32 (2014) 155–168

Contents lists available at ScienceDirect

Image and Vision Computing

journal homepage: www.elsevier.com/locate/imavis

Editor's Choice Article

Euclidean upgrading from segment lengths: DLT-like algorithm and its variants ☆

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ARTICLE INFO

Article history:
Received 13 March 2013
Received in revised form 7 December 2013
Accepted 30 December 2013

Keywords:
DLT-like algorithm
Euclidean upgrading
Multi-camera calibration
Segment lengths
Weighted DLT-like algorithm

ABSTRACT

In this paper, how to calibrate a fixed multi-camera system and simultaneously achieve a Euclidean reconstruction from a set of segments is addressed. It is well known that only a projective reconstruction could be achieved without any prior information. Here, the known segment lengths are exploited to upgrade the projective reconstruction to a Euclidean reconstruction and simultaneously calibrate the intrinsic and extrinsic camera parameters. At first, a DLT(Direct Linear Transformation)-like algorithm for the Euclidean upgrading from segment lengths is derived in a very simple way. Although the intermediate results in the DLT-like algorithm are essentially equivalent to the quadric of segments (QoS), the DLT-like algorithm is of higher accuracy than the existing linear algorithms derived from the QoS because of a more accurate way to extract the plane at infinity from the intermediate results. Then, to further improve the accuracy of Euclidean upgrading, two weighted DLT-like algorithms are presented by weighting the linear constraint equations in the original DLT-like algorithm. Finally, using the results of these linear algorithms as the initial values, a new weighted nonlinear algorithm for Euclidean upgrading is explored to recover the Euclidean structure more accurately. Extensive experimental results on both the synthetic data and the real image data demonstrate the effectiveness of our proposed algorithms in Euclidean upgrading and multi-camera calibration.

1. Introduction

With an uncalibrated multi-camera system, a projective reconstruction of the 3D points in the common field of view is possible to be obtained from the correspondence between their projections. Rectifying this projective reconstruction to a Euclidean reconstruction, i.e. performing Euclidean upgrading, is equivalent to calibrating the intrinsic and extrinsic parameters of the cameras. In literature, camera calibration methods can be divided into two categories: self-calibration and reference-object-based calibration. The self-calibration methods do not use any geometry information about the scene and they usually require some sort of constraints on the intrinsic parameters [1–6], or constraints on the motion of cameras [7–9]. The reference-object-based methods are mainly based on some calibration objects of known size, such as 3D calibration object [10,11], 2D calibration object [12,13], and 1D calibration object [14–17]. Compared with the self-calibration methods, the reference-object-based methods can provide higher calibration accuracy due to the use of Euclidean knowledge.

In the case of calibrating multi-camera systems [18,19], both 3D calibration objects and 2D calibration objects are prone to be self-occluded so that the marker points on them cannot be observed by all the referred cameras simultaneously, whereas 1D calibration objects are immune to self-occlusion. In [14,16,17], the 1D objects contain at least three marker points in order to calibrate one or more cameras, where the marker points are required to be exactly collinear and the distances between these marker points need to be measured with high precision. Liebowitz and Carlsson [20] proposed a method to recover the Euclidean structure from an affinely distorted space by the knowledge of segment lengths. This method can be used to calibrate multiple affine cameras with a segment undergoing general motions. This calibration object, a segment with two end points, is more flexible than the conventional 1D objects, since only the distance between the two end points needs to be known, and the collinearity requirement is not involved anymore. Moreover, even without the knowledge about the segment length, a Euclidean reconstruction can still be obtained despite that its global scale is undetermined.

Can the Euclidean structure be recovered from a projectively distorted space from the knowledge of segment lengths? Ronda and Valdés [21] gave this question an affirmative answer and proposed three algorithms based on the quadric of segments (QoS) defined in a higher-dimensional space by a set of segments of fixed length. These algorithms based on the QoS need to solve a set of

☆ Editor's Choice Articles are invited and handled by a select rotating 12 member Editorial Board committee. This paper has been recommended for acceptance by Peter Sturm.

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http://dx.doi.org/10.1016/j.imavis.2013.12.012

homogeneous linear equations in 55 variables, and thus require at least 54 segments in the projective space. The work of Ronda and Valdés [21] is very interesting but involves some very complex mathematics. The main contributions of them are that the relation of the QoS with the standard geometry associated to the Euclidean structure of space is given and the explicit formulae are derived to obtain the dual absolute quadric and the absolute quadratic complex from the QoS.

In this paper, we find that for the Euclidean upgrading from segment lengths, it is not necessary to use such complex mathematics as in [21], or in other words, a DLT (Direct Linear Transformation)-like algorithm can be derived in a very simple way. The derived algorithm consists of two main steps: First, a set of 4-order polynomial equations on the plane at infinity and the image of the absolute conic (IAC) are introduced from a given projective reconstruction of the scene consisting of segments with known lengths. Then, these 4-order polynomial equations are transformed into a set of inhomogeneous linear equations in 54 variables by a simple linearization technique, from which the least squares solution to the 54 variables can be computed in a linear way. Although the intermediate results in the DLT-like algorithm are essentially equivalent to the QoS, the DLT-like algorithm extracts the plane at infinity more accurately than the existing linear algorithms derived from the QoS and thus has higher accuracy in Euclidean upgrading. In addition, considering that the constraint equations in the above DLT-like algorithm have different reliabilities due to the measurement errors, we design two strategies to assign weights on these constraint equations, resulting in two weighted DLT-like algorithms. At last, a new weighted nonlinear algorithm for Euclidean upgrading is explored to refine the obtained results by these linear algorithms.

The rest of this paper is organized as follows. In Section 2 we describe the notations and related work. Section 3 presents a DLT-like algorithm for Euclidean upgrading from segment lengths and discusses the relationship between the DLT-like algorithm and existing algorithms. Then, two weighted DLT-like algorithms are proposed in Section 4. Section 5 introduces a weighted nonlinear algorithm to refine the results by these linear algorithms. The experimental results are reported in Section 6, followed by some concluding marks in Section 7.

2. Preliminaries

2.1. Notations

Here, the homogeneous coordinates of a space point are denoted by $\mathbf{X} = [x_1, x_2, x_3, 1]^{\mathrm{T}}$, and the corresponding inhomogeneous coordinates are denoted by $\widetilde{\mathbf{X}} = [x_1, x_2, x_3]^{\mathrm{T}}$. A segment is determined by its two end points $\{\mathbf{X},\mathbf{Y}\}$. Given the images of m ($m \geq 4$) segments in general position under n ($n \geq 2$) views, the projective camera matrices and the projective reconstruction of the segments can be obtained from the correspondences between the images of the $2m$ end points [5], which are related to the real Euclidean structure by a 4×4 transformation matrix. The projective camera matrices of the n views are denoted by P_j ($1 \leq j \leq n$), and the projective reconstruction of the end points of segments $\{\mathbf{X}_i,\mathbf{Y}_i\}$ ($1 \leq i \leq m$) are denoted by $\mathbf{X}_{pi} = \left(\widetilde{\mathbf{X}}_{pi}^{\mathrm{T}}, 1\right)^{\mathrm{T}}$ and $\mathbf{Y}_{pi} = \left(\widetilde{\mathbf{Y}}_{pi}^{\mathrm{T}}, 1\right)^{\mathrm{T}}$.

2.2. Linear algorithms derived from the QoS

Ronda and Valdés [21] proposed three algorithms for Euclidean upgrading with a set of segments of known lengths. These algorithms are designed based on the quadric of segments (QoS): For segments $\{\mathbf{X},\mathbf{Y}\}$, let $\sigma(\mathbf{X},\mathbf{Y})$ be a 10-vector composed of the elements of the symmetric matrix $(\mathbf{X}\mathbf{Y}^{\mathrm{T}} + \mathbf{Y}\mathbf{X}^{\mathrm{T}})$. Then, for all the segments of fixed length d, $\sigma(\mathbf{X},\mathbf{Y})$ lie on a quadric called the QoS of length d. The QoS can be written as $\mathrm{C}_1 + \frac{d^2}{2}\mathrm{C}_2$, where the dimensions of the linear spaces spanned by C_1 and C_2 are respectively 20 and 35. Given a segment $\{\mathbf{X}_i,\mathbf{Y}_i\}$ of length d_i, the constraint equation on C_1 and C_2 is obtained as

$$\sigma^{\mathrm{T}}\left(\mathbf{X}_{pi}, \mathbf{Y}_{pi}\right)\left(\mathrm{C}_1 + \frac{d_i^2}{2}\mathrm{C}_2\right)\sigma\left(\mathbf{X}_{pi}, \mathbf{Y}_{pi}\right) = 0. \tag{1}$$

Given at least 54 segments of known lengths, C_1 and C_2 can be determined uniquely. Then, C_1 and C_2 are used respectively to extract the Euclidean structure in the following three algorithms (C1S, C1A, and C2A):

2.2.1. C1S and C1A

C_1 in Eq. (1) determines the real structure up to a Euclidean transformation with an unknown scale. An explicit formula to derive the absolute quadratic complex from C_1 is given in [21], where the absolute quadratic complex is the quadric in terms of the Plücker coordinates of the lines which intersect the absolute conic [6]. A rectifying matrix is then extracted from the absolute quadratic complex to recover a Euclidean reconstruction [6], where the scale ambiguity can be determined by changing the overall scale of the structure such that the length constraints are satisfied. This algorithm is called C1S.

The scale ambiguity of the Euclidean reconstruction can also be reduced with the affine adjustment algorithm [20]. Here, an affine transformation matrix is computed linearly to minimize the total reconstruction error of segment lengths. The algorithm C1S followed by the affine adjustment step is called C1A. And the experimental results in [21] show that the affine adjustment step can improve the accuracy of Euclidean upgrading.

2.2.2. C2A

C_2 in Eq. (1) encodes the information of the plane at infinity. In the implementation of the algorithm C2A in [21], the plane at infinity is computed as the polar plane of the center of a sphere with respect to the sphere. For a given space point $\mathbf{Y}$, the sphere $\mathrm{A}_{\mathbf{Y}}$ with center $\mathbf{Y}$ and radius infinity is extracted from C_2 through

$$(\mathrm{A}_{\mathbf{Y}})_{ij} = \sigma^{\mathrm{T}}(\mathbf{e}_i, \mathbf{Y})\mathrm{C}_2\sigma\left(\mathbf{e}_j, \mathbf{Y}\right),$$

where $(\mathrm{A}_{\mathbf{Y}})_{ij}$ is the element in the i-th row and the j-th column of $\mathrm{A}_{\mathbf{Y}}$, and $\mathbf{e}_i$ is the i-th column of the 4×4 identity matrix. And then the plane at infinity is obtained as the polar plane of $\mathbf{Y}$ with respect to $\mathrm{A}_{\mathbf{Y}}$, i.e.

$$\pi_\infty = \mathrm{A}_{\mathbf{Y}}\mathbf{Y}. \tag{2}$$

When C_2 is not accurate due to the error in the projective reconstruction of the segments, the result of Eq. (2) is dependent on the choice of $\mathbf{Y}$. Therefore, multiple values of $\mathbf{Y}$ are used to compute a matrix which is composed of multiple estimations of the plane at infinity and then a final result is obtained by the SVD of this matrix. Next, a rectifying matrix is constructed from the plane at infinity to transform the projective reconstruction to an affine reconstruction. At last, the affine adjustment [20] is performed to refine the affine reconstruction to a Euclidean reconstruction.

A simpler way to directly extract the plane at infinity from C_2 was also given in [21]. In this simpler way, the plane at infinity is computed as

$$\pi_\infty = \begin{pmatrix} \sigma^{\mathrm{T}}(\mathbf{e}_1, \mathbf{e}_\alpha)\mathrm{C}_2\sigma\left(\mathbf{e}_\beta, \mathbf{e}_\gamma\right) \\ \vdots \\ \sigma^{\mathrm{T}}(\mathbf{e}_4, \mathbf{e}_\alpha)\mathrm{C}_2\sigma\left(\mathbf{e}_\beta, \mathbf{e}_\gamma\right) \end{pmatrix}, \tag{3}$$