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ScienceDirect

Information Fusion 8 (2007) 40–55

INFORMATION FUSION

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Integrity-directed sequential state estimation: Assessing high reliability requirements via safe confidence intervals ☆

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Received 1 November 2004; received in revised form 1 December 2005; accepted 2 December 2005
Available online 28 February 2006

Abstract

This study deals with the problem of dynamic state estimation of continuous-time systems from discrete-time measurements in the context of high-integrity applications. The objective of integrity-directed estimation is to provide confidence intervals for the state with an extremely low risk of error. We suppose that the process noise can be modelled by Gaussian sequences, and that the measurement noise is Gaussian in the normal operating mode of the sensors. The evolution of the posterior probability distribution of the system's state is deduced from recursive linear MMSE estimation. The estimation scheme presented here is equivalent to the Kalman filter, with the difference that the data is not processed directly, but collected in sets in preparation for an ulterior, slightly delayed, grouped processing. This strategy is particularly suitable for fault detection, because the estimator naturally takes into account the cross-correlations of close-in-time measurements and the decisions can be based on more data. Next, we introduce dynamic tools for detecting faults and sensor failures. A full Bayesian modelling of the sensors leads to the derivation of a dynamic multiple-model estimator performing the linear MMSE state estimation under various fault hypotheses. This estimator provides estimates of the posterior density function of the state, on which safe confidence intervals certifying very high-integrity levels can be fixed. In practice, the exponential growth of the complexity of the multiple-model estimator requires the simplification of the posterior mixture distributions. A new method for limiting the complexity of the posterior distributions in an integrity-oriented context is presented. Unlike the known mixture simplification strategies (GPB, IMM), the present method has the property to quantify and minimize the loss in integrity of the process of simplification of the distributions. Finally, the estimator is tested on a typical rail navigation problem.

Keywords: Integrity; Dynamic estimation; Fault detection; Safe navigation systems; Rail navigation; Robust estimation; Bayesian framework; Dynamic multiple-model estimator; Gaussian mixture; Kullback–Leibler distance; Overbounding; Markov chains; Odometry; Kalman filter

1. Introduction

The present study is motivated by the increasing demand for automatisation of transport systems in today's society. Various levels of application are concerned, ranging from aid to decision to automatic traffic control. The decision and control processes of most navigation systems, such as those in use in current railway systems, rely on the estimation of the positions and speeds of the mobiles, carried out for instance with onboard sensors. In the framework of automatic traffic control, it is vital to ensure the safety of the state estimation procedure, in the sense that the estimation algorithms must provide a region of space where the mobile lies, and this with an extremely low risk of mistake. This study deals with the problem of dynamic state estimation of continuous-time systems from discrete-time measurements in the context of applications requiring very high levels of integrity. The problem of integrity-directed estimation is seldom considered in the

☆ A short version of this study was presented at the Seventh International Conference on Information Fusion (FUSION 2004) in Stockholm, Sweden [1].

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1566-2535/$ - see front matter
doi:10.1016/j.inffus.2005.12.001

literature. It is well known that the linear minimum mean square error (MMSE) estimator is optimum in the context of linear dynamics and Gaussian variables. Unfortunately, real sensors are often biased and may provide erroneous measurements (*outliers*), causing important biases in the linear estimation. As a consequence, the reliability of the estimation may drop considerably. This issue could have dramatic consequences in applications where the integrity of the estimates is of first importance. Unlike linear estimators, *robust* estimators present a reduced sensitivity to erroneous measurements, and tend to be more powerful in practical applications.

The objectives of integrity-directed estimation differ from those of robust estimation. The goal of integrity-directed estimation is to provide extremely safe confidence intervals for the actual value of the system's state, rather than an accurate estimate of the state. Robust estimation, on the other hand, aims at deriving estimators that perform well not only in 'normal conditions', but also in the least favourable conditions of noise. The performance criterion in robust estimation is generally a *statistical accuracy* criterion, such as the minimum mean square error criterion or derivatives. The robust estimation literature is quite vast, although few papers are really integrity-directed. We owe the work of reference in robust estimation to Huber, who established the basics of static robust estimation in [2]. Huber's approach of the problem of seeking robust estimators relies on the *minimax* strategy. The so-called *optimal estimator* is the one with the best performance under the least favorable noise distribution, the performance criterion being the minimum asymptotic covariance of the estimation error when the amount of measurements grows to infinity. In dynamic estimation however, the asymptotic performance criterion makes little sense because the variables to be estimated are randomly changing over time. Moreover, an accuracy criterion based on the expected variance of the estimation error is highly insufficient in the context of applications demanding very high levels of confidence on the state estimates. For instance, the safe navigation systems according to the European Norm Standards (e.g. EN50126) require *Safety Integrity Levels* (SIL) of only 10^{-7} (SIL3) or even 10^{-9} (SIL4) error per hour. Extending the ideas of Huber to the robust dynamic estimation problem, Mangoubi studied in [3] the stability and the performance of dynamic estimators with respect to unknown perturbations of the model's dynamics and the measurement noise. Mangoubi derives the dynamic state estimators that minimize the impact of those perturbations on the estimate and provides statistics on the estimation error. Schick and Mitter presented in [4] a multiple-model dynamic state-estimator (similar to the estimator developed here) which is robust in the presence of rare and isolated outliers and ensures the minimum estimation error variance. The context of this study is different since in safety applications the integrity of the state estimation prevails over its precision. The objective is no longer to minimize any cost function related to the accuracy of the estimates but to provide safe confidence intervals.

The present article is inspired from the work of Ober, who proposed in [5] a complete solution to the static integrity-directed estimation problem for navigation systems. Ober's approach consists roughly in using fault detection techniques to isolate the faulty sensors, providing a nonlinear estimation from the unbiased measurements only and associating with the state estimate a safe confidence interval which certifies a very high level of integrity. This study represents a continuation of [1], which aimed at showing how those static robust estimation techniques could be adapted to the dynamic problem. In addition, Ober proposed a solution for the robust dynamic estimation problem which consisted of a robust version of the Kalman filter (in the sense of Huber) based on the results of Kovacevic [6]. The *robust Kalman filter* considers the prior estimates and the innovations as the inputs of weighted least-squares estimation problems, and uses Huber's M-estimator to deduce the posterior estimates in an automated way. Unfortunately, the robust Kalman filter does not provide confidence intervals for the estimates, nor does it certify the integrity of those estimates. Unlike most approaches of the state estimation problem, which simply provide an estimate of the state optimal according to some precision criterion, we try to estimate the full probability distribution of the state. Thus this study is concerned with *probability density function estimation* more than typical state estimation. Recent work in dynamic estimation [7] presents the sequential Monte Carlo (SMC) methods and particle filters as an efficient solution to the dynamic problem of fault detection and nonlinear density estimation. Those methods derive an approximation of the posterior distribution of interest by simulating the evolution of samples of the density function [8]. The SMC methods are known to reveal the main lobes and local extrema of the posterior density functions. However, the relative precision in the tails of the posterior densities depends directly on the quality of the sampling. In our opinion, the amount of particles and computation volume needed to achieve the integrity level SIL4 with SMC methods is unmanageable. The density estimation method presented in this article is based on the decomposition of the density function in Gaussian components rather than particles. In this study, we suppose that the process noise of the system and the measurement noises of the sensors in their normal operating mode follow Gaussian distributions. Gaussian distributions, which are fully defined by their first two moments, are intensively used in the engineering literature. The choice of Gaussian distributions is motivated by the central limit theorem on the one hand, and the recently developed techniques of overbounding on the other hand. Overbounding aims at replacing the actual distributions of the variables by safe Gaussian pseudo-distributions that still guarantee the final integrity of the estimation process [9].

We use the recursive linear MMSE estimator, also called *Kalman filter*, as a basic Gaussian density estimation tool. In Section 2, we explain the discretization process of the continuous-time system, and show how the equations of