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An improved differential box-counting method to estimate fractal dimensions of gray-level images

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ABSTRACT

The differential box-counting (DBC) method is one of the frequently used techniques to estimate the fractal dimension (FD) of a 2D gray-level image. This paper presents an improved DBC method based on the original one for improvement of the accuracy. By adopting the modifying box-counting mechanism, shifting box blocks in (x, y) plane and selecting appropriate grid box sizes, it can solve the two kinds of problems which the DBC has: over-counting boxes along z direction and under-counting boxes just at the border of two neighboring box blocks where there is a sharp gray-level abruption exits. The experiments using two sets of synthetic images and one set of real natural texture images demonstrate that the improved DBC method can solve the two kinds of problems perfectly, simultaneously, and can outperform other DBC methods in the accuracy.

1. Introduction

Most of the objects in the real world are so complex and irregular in nature that they cannot be described by classical Euclidean geometry [1–4]. Fractals, initially developed by Mandelbrot [5], are mathematical sets with high degrees of geometrical complexity that can model many natural phenomena. In fractal theory, a fractal is defined as a set for which the Hausdorff–Besicovich dimension (simply, Hausdorff dimension) is greater than the topological (Euclidean) dimension [1,5]. One of the fundamental properties of a fractal is its self-similarity. The fractal dimension (FD) is a useful numerical quantity for expressing the self-similarity of a fractal. Quite a few different methods have been proposed to estimate the FD. Pentland [6] proposed a method of estimating FD by using the Fourier power spectrum of image intensity surfaces. Peleg et al. [7] adopted Mandelbrot's idea of the ε-blanket method and extended it to surface area calculation. Clarke [8] used the concept of triangular prism surface area to compute the FD. Gangepain and Roques-Carmes [9] developed the popular reticular cell-counting method which improved upon by Voss [10] by incorporating probability theory. Later, Keller et al. [11,12] contributed a further refinement by a way of linear interpolation. Lopes and Betrouni [13] summarized and classified those methods into three major categories: the box-counting (BC) methods, the fractional Brownian motion (fBm) methods and the area measurement methods.

The box-counting dimension calculated by the box-counting methods is the most frequently used method for measurements in various application fields. The reason for its dominance lies in its simplicity and automatic computability [14]. To the gray-level images, the differential box-counting (DBC) method proposed by Sarkar and Chaudhuri [15–17] was considered as a better one, as was also supported by the investigation in many literatures. It is a generalization of classical BC method to estimate the FD of gray-level images [18–22].

However, there are some drawbacks in the original DBC method. Both Chen et al. [1] and Li et al. [19] pointed the shortcomings of over-counting or under-counting the number of boxes. Later many modified or improved DBC methods have been proposed for those problems. Jin et al. [22] described a relative differential box-counting (RDBC) method by using a practical algorithm with the upper and lower limits for estimating FD. They showed that their method is better with smaller distance error and covers the full dynamic range of FD values. Biswas et al. [21] proposed the modified implementation of DBC by adopting a parallel algorithm to make its computability more efficient. In the shifting differential box-counting (SDBC) method by Chen et al. [1], the concept of shift operation is similar with that in RDBC. However, the authors also pointed that the SDBC cannot work to

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the images in which a sharp gray-scaled variation exists at the border of the neighboring box blocks. Li et al. [19] proposed an improved DBC. Three modifications were carried out for better performance of estimating FD in their method. But the images with a sharp gray-scaled variation were not used in their experiments.

Base on the above summary, we can find that, so far, there is no one improved DBC algorithm which can deal with the two problems of over-counting and under-counting the number of boxes, simultaneously. According to our experience on the analysis of gray level images, the two drawbacks in the DBC method may provide unreasonable FD values which may be less than two, particularly to the images with a sharp gray-scaled variation at the border of the neighboring box blocks. Therefore, for computing FD more accurate, it will be necessary and significant meaningful to improve a method which can deal with the two problems, simultaneously. In this paper, an improved DBC method has been proposed to improve the accuracy of estimating FD. In our improved method, the modifying box-counting mechanism, shifting box blocks in (x, y) plane and selecting appropriate grid box sizes have been implemented. The experiments results using the two set of synthesized images and one set of texture images demonstrated the advantages of our method.

The remainder of this research is organized as follows. Section 2 is the basic definition of FD and DBC method. The drawbacks of DBC and two kinds of problems are discussed as well. Section 3 describes our improved method with three improvements, and the procedure. Section 4 gives results and discussions of the three experiments. Section 5 draws the conclusions.

2. Basic definition and different DBC approaches

2.1. Basic fractal dimension definition

Mandelbrot [5] defined a fractal as a bounded set A in R for which the Hausdorff–Besicovich dimension is strictly larger than the topological dimension. Consider a bounded set A in Euclidean n-space. The set is said to be self-similar if A is the union of $N_r(A)$ distinct (non-overlapping) subsets, each of which is similar to A scaled down by a ratio r. Thanks to the celebrated box-counting theorem [23], fractal dimension, D, of A can be rewritten in the form

$$1 = N_r(A) \cdot r^D \quad \text{or} \quad D = \frac{\ln N_r(A)}{\ln(1/r)}. \tag{1}$$

The D can only be calculated for deterministic fractals. For an object with deterministic self-similarity, its FD is equal to its box-counting dimension D_B. However, natural scenes practically do not exhibit deterministic self-similarity. Therefore, in general, it is hard to directly compute the exact value of D. Fortunately; those natural scenes exhibit some statistical self-similarity. The box-counting dimension D_B is an estimate of D, which is calculated by using a box-counting method. To calculate the FD of a gray scale image, the DBC method is the most frequently used algorithm which is introduced as follow.

2.2. DBC and drawbacks

The method is based on the reticular cell counting approach proposed by Gangepain and Roques-Carmes [9]. The reticular cell counting approach treats a box which contains at least one sample of gray-level intensity surface as a countable box. And then perform the least square linear fit of $\log N_L$ versus $\log L$ to obtain the D, where the L is the box size, N_L is the number of boxes needed to cover the whole gray-level intensity surface. Sarkar and Chaudhuri [16] adopted a differential method to count the N_L, so they call their method as the *differential box-counting* approach (DBC). Detail description is listed below.

Consider an image of size $M \times N$ as a three-dimensional (3D) spatial surface with (x, y) denoting pixel position on the image plane, and the third coordinate (z) denoting pixel gray-level. In the DBC method, the (x, y) plane is partitioned into non-overlapping blocks of size $s \times s$, where $M/2 \geqslant s \geqslant 2$ and s is an integer. Then let an estimate of $r = s/M$. On each block, there is a column of boxes of size $s \times s \times s'$, where s' is the height of each box, $G/s' = M/s$ and G is the total number of gray-levels. Assign numbers $1, 2, \ldots$ to the boxes as shown in Fig. 1. Let the minimum and maximum gray-level (write as $I_{\min}$ and $I_{\max}$) in the (i, j)th block fall in box number k and l, respectively. The boxes covering this block are counted in the number as

$$n_r(i,j) = l - k + 1, \tag{2}$$

where the subscript r denotes the result using the scale r.

For example, $s = s = 3$ and $n_r(i, j) = 3–1 + 1$ as illustrated in Fig. 1. Considering contributions from all blocks, N_r is counted for different values of r as

$$N_r = \sum n_r(i,j). \tag{3}$$

Then the FD can be estimated from the least squares linear fit of $\log(N_r)$ versus $\log(1/r)$.

Sarkar and Chaudhuri [16] compared their method with other four algorithms in their paper. They showed DBC have a lower computational complexity, best in terms of efficiency and dynamic range of FDs, but lower accuracy. However, some disadvantages of DBC have been discussed in other literatures [1,19]. They pointed that the DBC method has two major problems.

(1) Over-counting the number of boxes covering the image intensity surface. It may occur in the z direction and in the x or y directions as well, if the boxes do not shift appropriately along those directions.
(2) In contrast, under-counting the number of boxes may happen at the border of the neighboring box blocks where there is a sharp gray-scaled variation exists on the image intensity surface.

Because of the above two drawbacks, unreasonable results will be obtained when using the method to measure the following images.

(1) Images with the same roughness of the intensity surface but different gray-levels. For simplify, if a new image is generated for which the gray-level of each pixel in the original image is either increasing or decreasing by a constant value, then the FD of the new image should stay the same as that of the original image theoretically since they have the same degree of roughness. But these two estimates appear to be distinct when the DBC method is adopted.
(2) Images which have sharp gray-level abruption just at the border of two neighboring box blocks (see detailed description in Section 4.2). The estimated FDs by the DBC method may lie outside the range between 2.0 and 3.0. These results, of course, are not reasonable.

To solve those problems and compute FD efficiently and accurately, some modified or improved DBC methods based on the original one have been proposed. Jin et al. [22] described a relative differential box-counting (RDBC) method by using a practical algorithm and setting the upper and lower limits of the scale for estimating FD. N_r is counted in their method as follows.

$$N_r = \sum ceil[d_r(i,j)/s'], \tag{4}$$