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An efficient unsupervised method for obtaining polygonal approximations of closed digital planar curves ☆

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ABSTRACT

The contour of a shape is a powerful feature that enables its description and subsequent recognition. However, the direct use of a contour introduces redundancy. Many algorithms have been proposed for simplification of a contour while its most outstanding features are maintained. However, several inconveniences can be found in these methods, mainly the need for user interaction to set proper values for the parameters and, in some cases, for each specific contour. The proposed algorithm obtains polygonal approximations of contours and does not have parameters that must be adjusted, which provides the best balance between fidelity and efficiency, and has a modest algorithmic complexity. The method is based on an analysis of the convexity/concavity tree of the contour, and an efficient split/merge strategy is used. The experiments conducted show that the proposed method overcomes the state-of-the-art, using both synthetic data and a broad dataset of real contours.

1. Introduction

The obtainment of polygonal approximations of digital planar curves is used for various activities, such as image analysis [1], shape analysis [2] and digital cartography [3].

Given that the contour of a shape is described by the sequence of points $\mathbf{C}:\{x_i,y_i\}, i=0,\dots,N-1$, where N defines the number of vertices of the contour, the problem is to find the subset $\mathbf{D}:\{x_i,y_i\}, i=0,\dots,M-1$, where $M \leqslant N$ and $\mathbf{D}\subset\mathbf{C}$. A desirable property of the polygonal approximation of $\mathbf{C}$ defined by $\mathbf{D}$ is to provide good values for fidelity and efficiency. The fidelity is a measure of the distortion introduced by the approximation relative to the original contour, and the efficiency is a measure of the simplification obtained by the approximation. Because these two objectives conflict, finding the best balance between them is complicated.

Many algorithms for polygonal approximation of digital planar curves have been proposed. In general, these algorithms can be classified according to the problem solved [4]:

- The problem *min*-ϵ: Fix the number of points M of the approximation, and look for the subset $\mathbf{D}\subset\mathbf{C}$ that defines the polygonal approximation of $\mathbf{C}$ with the least possible distortion, or alternatively, what is the polygonal approximation with size M that provides the highest fidelity?
- The problem *min*-#: Fix the maximum allowed distortion value for the polygonal approximation, and find the polygonal approximation that has the smallest number of points M, or alternatively, what is the polygonal approximation given the maximum allowable error ϵ that provides the highest efficiency?

Some algorithms have been described to find the optimal polygonal approximation for both problems (see a list of them in [5]). The main problem of the optimal methods is that their algorithmic complexity is very high. For example, the known method based on dynamic programming [6] using the L_2 metric to solve the *min*-ϵ problem has a complexity of $O(MN^2)$ when only a starting point is used (in other words, to open the curves) and $O(MN^3)$ when testing every point of a closed curve as the starting point. Although this problem could be reduced if a parallelised version of the algorithm is executed in an efficient parallel architecture, such as those proposed in [7,8], the high algorithmic complexity of the optimal methods has motivated many researchers to look for sub-optimal methods that have lower complexity.

In this paper, we propose a new sub-optimal algorithm for efficiently generating a polygonal approximation of a contour. This new algorithm has no parameters and generates, in an unsuper-

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vised manner, the polygonal approximation that has the best balance between fidelity and efficiency.

The remainder of this paper is structured as follows: Section 2 presents the related work. Section 3 describes the proposed method. Section 4 describes the experiments conducted and the results obtained. Section 5 discusses some of the relevant aspects of the proposed method and experimental results. Finally, Section 6 shows the main conclusions.

2. Related work

In general, suboptimal methods attempt to find the smallest number of characteristic points that define the shape. These points are known as dominant points, and many different methods for detecting these points have been proposed in the literature. One of the simplest methods is to directly use the contour breakpoints (points where the chain code changes). However, in general, due to the digitisation process and noise, using breakpoints directly introduces redundancy. Thus, the methods that use the set of breakpoints usually propose a refining step to filter the noise in this set.

Another alternative to using breakpoints was proposed in a previous work of the authors [9]. In this work, the use of an extended concavity–convexity tree of a contour to generate a first set of candidate points to be the dominant points is proposed. Then, the set of candidate points thus obtained is used as the initial set of points for two well-known methods [10,11], which iteratively refine this set to generate a final polygonal approximation. The main conclusions of this work were the new way of generating an initial set of candidate points, which allows a significant reduction in the computation times and improves the quality of the approximations obtained.

Another important issue is to find the correct answer to the following question: How many linear segments are sufficient to represent a shape? Alternatively, what is the polygonal approximation that provides the best balance between fidelity and efficiency?

To answer these questions, most optimal and suboptimal methods opt for a supervised approach in which the user must set the value of at least one parameter, the values of the points used by the approximation (M) or the value of the maximum allowable distortion of the approximation with regard to the contour (ϵ). In general, the appropriate value for these parameters to provide a good balance between efficiency and accuracy depends on each specific contour and, as a consequence, is not easy to obtain.

In two previous studies by the authors [12,13], several versions of the measure known as Figure-Of-Merit (FOM) have been used as a cost function in order, to answer this question in an automatic way. This measure was first proposed in [14] as a quality measure provided by a polygonal approximation, and it can be expressed as

$$\text{FOM} = \frac{\text{CR}}{\text{ISE}}, \tag{1}$$

where CR is the compress ratio (N/M), and ISE represents the integrated square error between the contour **C** and the approximation **D**. The ISE is used similar to a distortion measure. See Section 3.2 for a formal definition.

Later, Rosin [15] showed that the measure FOM is biased. For this reason, a modified measure has been used, where the factor M is given greater weight to correct the bias. Carmona-Poyato et al. [12] studied which of the following versions of FOM, when they are used as a cost function to be optimised, provides the best balance between fidelity and efficiency:

$$\text{FOM-1} = \text{ISE} \cdot M, \tag{2}$$

$$\text{FOM-2} = \text{ISE} \cdot M^2, \tag{3}$$

and

$$\text{FOM-3} = \text{ISE} \cdot M^3. \tag{4}$$

These measures were originally proposed by Marji et al. [4] for quality evaluation.

Another disadvantage of the suboptimal methods is their dependence on a parameter that defines the stop condition. The problem is that the value of this parameter must be given in a supervised manner. However, Prasad et al. [16] have proposed a framework based on obtaining the maximum admitted deviation (due to the digitising process) of a straight segment relative to the contour points that are approximated by that segment.

The method proposed in this paper is based on an efficient strategy of split/merge that takes advantage of the combination of the previous results obtained by the authors [9,13] and the framework proposed by Prasad et al. Thus, a new algorithm has been designed that addresses several of the above issues: (1) it is unsupervised and, as such, provides a good balance between fidelity and efficiency; (2) it has no parameter to be set; and, finally, (3) it uses an efficient split/merge strategy that has moderate algorithmic complexity.

3. Proposed method

Our proposal is an overall method in the sense that it searches for a polygonal approximation that provides the best balance between fidelity and efficiency. This goal is accomplished by obtaining the approximation that provides the best value of the cost function FOM-2. Moreover, the proposed method can also be easily modified to solve the problems *min-#* or *min-*ϵ separately.

The method uses an efficient strategy of Split–Merge to conduct the search mentioned above. The split stage is used to locate a small set of candidate points. Section 3.1 explains this stage in more detail.

Subsequently, the merge stage is used to find, starting from the reduced set of candidate points generated by the previous split stage, the contour points that define the polygonal approximation with the best value of the cost function FOM-2. Section 3.2 explains this stage in more detail.

3.1. Split stage

Several strategies have been used to select an initial set of contour points from which a polygonal approximation is obtained. For example, some authors begin by directly using the break points (points of a contour whose chain code changes from the previous point in the sequence) [11,10]. However, if the break points are used as the initial set, then many more than the necessary number of iterations could be performed in the subsequent merge stage. In a previous paper [9], the authors proposed to use a method that performs an analysis of the convexity/concavity of the contour to obtain a reduced set of candidate points. To accomplish this goal, the convex hull of a set of points is used. The convex hull of a set **X** of points in the Euclidean plane or Euclidean space is the smallest convex set that contains **X**. The convex hull has been previously used, for example, in [17], to speed up the known method of Douglas et al. [3]. In [9], the convex hull is used in a different spirit, as will be shown below.

In the first iteration, the convex hull of the original contour is obtained, and this set is the initial set of candidate vertices that define the initial polygonal approximation.

Then, each segment of the initial polygonal approximation is checked as to whether the local value of the criterion FOM1 (2) associated with that segment could be improved by the approximation obtained from the convex hull of the contour points that