J. Vis. Commun. Image R. 39 (2016) 172–180

Contents lists available at ScienceDirect

J. Vis. Commun. Image R.

journal homepage: www.elsevier.com/locate/jvci

Speckle noise removal via nonlocal low-rank regularization ☆

Yulian Wu *

Department of Health Management, Xi'an Medical University, Xi'an 710021, China
School of Science, Xidian University, Xi'an 710071, China

ARTICLE INFO

Article history:
Received 24 September 2014
Revised 26 April 2016
Accepted 29 April 2016
Available online 4 May 2016

Keywords:
Speckle noise
Nonconvex optimization
Low-rank approximation
Augmented Lagrange multiplier
Structured sparsity

ABSTRACT

This paper presents a novel method for speckle noise removal. We propose a nonlocal low-rank regularization (NLR) approach toward exploiting structured sparsity and explore its application into speckle noise removal. A nonconvex surrogate functions for the rank instead of the convex nuclear norm is proposed. To further improve the computational efficiency of the proposed algorithm, we have developed a fast implementation using augmented Lagrange multiplier (ALM) method. We experimentally demonstrate the excellent performance of the technique, in terms of both Peak Signal to Noise Ratio (PSNR) and Structural Similarity (SSIM).

1. Introduction

Images generated by coherent imaging modalities, e.g. synthetic aperture radar (SAR), ultrasound and laser imaging, inevitably come with multiplicative noise (also known as speckle), due to the coherent nature of the scattering phenomena. The speckle noise seriously interferes with the upper tasks, such as object recognition [1] and image segmentation [2]. Due to the coherent nature of the image acquisition process, in the speckle noise models, the noise field is multiplied by (not added to) the original image, and it is described by a non-Gaussian probability density function, with Rayleigh and Gamma being common models [3]. So it is signal independent, non-Gaussian, and spatially dependent. Hence speckle denoising is a very challenging problem compared with additive Gaussian noise.

Speckle noise removal methods have been discussed in many references. Since exploiting a prior knowledge of the original images (e.g., sparsity) is critical to the success of speckle noise removal. Popular methods include bilateral filtering for despeckling [4], wavelet based despeckling approaches [5], TV-based variational approaches [7–11]. The first total variation-based speckle noise removal model (RLO-model) was presented by Rudin et al. [7], which used a constrained optimization approach with two Lagrange multipliers. Aubert and Aujol [8] proposed their speckle noise removal model (AA model) in the framework of the maximum a posteriori probability (MAP) estimation. Because of the non-convexity of AA model the global solution is hard to find. To resolve this problem, Bioucas-Dias and Figueiredo [10] applied the MAP estimation method in the log domain and proposed a convex speckle noise removal model (BF model). In addition, Steidl and Teuber [11] also derived a convex model (ST model), in which the I-divergence was used as the fidelity term. Indeed, solutions of variational problems with TV regularization admit many desirable properties, most notably the appearance of sharp edges. However, the regularization with TV also has so-called staircasing artifacts in the smooth image regions. To overcome the drawback, the total generalized variation (TGV) regularizer [12] also has been investigated in a recent work (TGVSNR model) [13], which incorporated the TGV penalty into the existing data fidelity term for speckle removal, and developed two novel variation despeckling models. TGV-based despeckling method outperforms the traditional TV methods by reducing the staircasing artifacts. Besides above convex variational approaches for speckle noise removal, Han et al. [17] applied nonconvex TV regularizer to the speckle noise removal (NRSNR model), which preserves edges better of restored images than classical TV regularizer-based methods. It is worth noting that Chen et al. proposed a FoE method based higher-order Markov Random Fields (MRF) [27].

Meanwhile, the more advanced denoising methods e.g., nonlocal mean (NLM) [6], block-matching 3D (BM3D) [14] and K-SVD [15] have been readily extended to SAR despeckling [16,18]. Parrilli et al. [18] derived a SAR-oriented version of BM3D. It exhibits an objective performance comparable or superior to other techniques

☆ This paper has been recommended for acceptance by Prof. M.T. Sun.

* Department of Health Management, Xi'an Medical University, Xi'an 710021, China.

E-mail address: wyl_wp711@163.com

http://dx.doi.org/10.1016/j.jvcir.2016.04.024

on simulated speckled images, and guarantees a very good subjective quality on real SAR images. As we all know that the nonlocal property of image patches and sparsity property of natural images are common. In [19] Dong et al. have shown an intrinsic relationship between simultaneous sparse coding (SSC) [20] and low-rank approximation. Based on the above considerations, we propose to regularize speckle noise removal by the method of patch grouping and low-rank approximation. Specifically, we group a set of similar image patches to form a data matrix Y for each exemplar image patch. Since these similar image patches contain similar structures, the rank of this data matrix Y is low meaning a useful image prior. To more efficiently solve the problem of rank minimization, we propose to use a nonconvex smooth surrogate function for the rank, which leads to an iterative singular-value thresholding. Experimental results on natural images show that our low-rank approach has the ability to achieve greater speckle noise removal (average PSNR gain over >1 dB) than recent TGV approach and nonconvex TV approach, further the performance on real SAR image is compared with SAR-BM3D [18] approach.

2. Related work and discussion

In this subsection, the major TV-based variational models and the corresponding algorithms are briefly reviewed.

Let $f \in R_+^n$ denote an n-pixel observed image, assumed to be a sample of a random image F, It is known that f can be assumed to be the product of the underlying true image intensity $z \in R_+^n$ and the speckle noise $\eta \in R_+^n$:

$$f_i = z_i \eta_i \quad for\ i = 1, \ldots, n \tag{1}$$

where η is (uncorrelated)multiplicative noise with unit mean, $E(\eta_i) = 1$, the probability density function of η for the L-look SAR image is given by the following Gamma distribution [25]:

$$p(\eta) = \frac{1}{\Gamma(L)} L^L \eta^{L-1} e^{-L\eta} \tag{2}$$

where L is a positive integer, Γ is the usual Gamma function.

2.1. Major TV-based variational models

The MAP criterion is applied to Eq. (1), Aubert and Aujol [8] derived a new model (AA-model):

$$z^* = \arg\min_z \left\{ L\sum_{i=1}^{N} (\log z_i + f_i z_i^{-1}) + \lambda \sum_{i=1}^{N} |\nabla z_i| \right\} \tag{3}$$

where z^* denotes the despeckled result, $\lambda > 0$ denotes the regularization parameter.

From the minimization problem (3), we can observe that the global solution of the AA model is hard to find because it has a nonconvex fidelity term. To resolve this problem, Authors in [10] take logarithmic transformation ($u_i = \log z_i$) to convert Eq. (1) into an additive form, using the MAP criterion, the restored image of the BF model can be inferred by solving the following variational problem:

$$u^* = \arg\min_u \left\{ L\sum_{i=1}^{N} (u_i + f_i e^{-u_i}) + \lambda \sum_{i=1}^{N} |\nabla u_i| \right\}$$
$$z^* = e^{u^*} \tag{4}$$

3. Problem formulation

In this section, we present a new model of nonlocal low rank regularization for speckle noise removal. The proposed regularization model consists of two components: patch grouping for characterizing self-similarity of images and low-rank approximation for sparsity restriction. The basic assumption underlying the proposed approach is that self-similarity is abundant in natural images. Such assumption implies that a sufficient number of similar patches can be found for any exemplar patch of size $\sqrt{s} \times \sqrt{s}$ at position i denoted by $\boldsymbol{x}_i$. For each exemplar patch $\boldsymbol{x}_i$, we perform a search of k-nearest-neighbor within a local window,

$$M_i = \{ i_j \| \boldsymbol{x}_i - \boldsymbol{x}_{i_j} \| < T \} \tag{5}$$

where T is a predefined threshold, and M_i denotes the collection of positions corresponding to those similar patches. After patch grouping, we obtain a data matrix $\boldsymbol{Y}_i = [\boldsymbol{x}_{i_0}, \boldsymbol{x}_{i_1}, \ldots, \boldsymbol{x}_{i_{m-1}}]$, $\boldsymbol{Y}_i \in C^{s \times m}$ for each exemplar patch $\boldsymbol{x}_i$, where each column of $\boldsymbol{Y}_i$ denotes a patch similar to $\boldsymbol{x}_i$.

Under the assumption that these image patches have similar structures, the formed data matrix $\boldsymbol{Y}_i$ has a low-rank property. In practice, $\boldsymbol{Y}_i$ may be corrupted by some noise, which could lead to the deviation from the desirable low-rank constraint. One possible solution is to model the data matrix $\boldsymbol{Y}_i$ as: $\boldsymbol{Y}_i = \boldsymbol{X}_i + \boldsymbol{W}_i, \boldsymbol{X}_i$ and $\boldsymbol{W}_i$ denote the low-rank matrix and the additive noise matrix respectively. Then the low-rank matrix $\boldsymbol{X}_i$ can be recovered by solving the following optimization problem:

$$\boldsymbol{X}_i = \arg\min_{\boldsymbol{X}_i} rank(\boldsymbol{X}_i), \quad \text{s.t. } \|\boldsymbol{Y}_i - \boldsymbol{X}_i\|_F^2 \leqslant \delta^2 \tag{6}$$

where $\|\cdot\|_F^2$ denotes the Frobenius norm and δ^2 denotes the variance of additive Gaussian noise. However, solving rank-minimization problem (6) is an NP-hard problem; hence we cannot solve Eq. (6) directly. Most previous works solve the convex surrogate of the rank problem for nuclear norm $\|\cdot\|_*$ instead. In practice, this constrained minimization problem can be solved in its Lagrangian form, namely

$$\boldsymbol{X}_i = \arg\min_{\boldsymbol{X}_i} \lambda \|\boldsymbol{X}_i\|_* + \|\boldsymbol{Y}_i - \boldsymbol{X}_i\|_F^2 \tag{7}$$

Eq. (6) is equivalent to Eq. (7) by choosing a proper λ.

It has been proved that under certain incoherence assumptions on the singular values of the matrix, solving the convex nuclear norm regularized problem leads to a near optimal low-rank solution [21]. In order to achieve a better approximation of the rank problem, nonconvex optimization toward rank minimization is proposed. The rank minimization problem (7) can be approximately solved by minimizing the following nonconvex function [22]:

$$X_i = \arg\min_{\boldsymbol{X}_i \in R^{s\times m}} \sum_{l=1}^{s} g_\lambda(\sigma_l(\boldsymbol{X}_i)) + ||\boldsymbol{Y}_i - \boldsymbol{X}_i||_F^2 \tag{8}$$

where $\sigma_l(\boldsymbol{X}_i)$ denotes the l-th singular value of $\boldsymbol{X}_i$. When $g_\lambda = \lambda x$, $\sum_{l=1}^{s} g_\lambda(\sigma_l(\boldsymbol{X}_i)) = \lambda \sum_{l=1}^{s} \sigma_l(\boldsymbol{X}_i) = \lambda \|\boldsymbol{X}_i\|_*$ .Without loss of generality, we assume $s < m$ in this work. The nonconvex penalty function $g_\lambda(x) = \lambda \log(|x| + \varepsilon)$ satisfies the following assumptions: $g_\lambda : R \to R^+$ is continuous, concave and monotonically increasing on $[0, +\infty)$. It is possibly non-smooth.

How to use the patch-based nonconvex low-rank regularization model for speckle noise removal? The basic idea is to enforce the nonconvex low rank property over the sets of nonlocal similar patches for each extracted exemplar patch along with convex fidelity term of model (4). With the proposed low-rank regularization term, we propose the following global objective functional for speckle noise removal:

$$\left(\hat{\boldsymbol{u}}, \hat{\boldsymbol{X}}_i\right) = \arg\min_{u, X_i} \left\{ L\sum_{i=1}^{n} (u_i + f_i e^{-u_i}) + \beta \sum_{i=1}^{n} \|\hat{R}_i \boldsymbol{u} - \boldsymbol{X}_i\|_F^2 + \sum_{i=1}^{n} \sum_{l=1}^{s} g_\lambda(\sigma_l(\boldsymbol{X}_i)) \right\} \tag{9}$$