Information Fusion 18 (2014) 20–32

Contents lists available at SciVerse ScienceDirect

Information Fusion

journal homepage: www.elsevier.com/locate/inffus

On the use of a multi-criteria approach for reliability estimation in belief function theory

Ahmed Frikha *

LOGIQ, High Institute of Industrial Management, University of Sfax – Tunisia, Route M'harza Km 1.5; B.P 1164, 3018 Sfax, Tunisia

ARTICLE INFO

Article history:
Received 11 March 2012
Received in revised form 27 March 2013
Accepted 1 April 2013
Available online 29 May 2013

Keywords:
Belief function theory
Information fusion
Multi-criteria decision aid
Expert judgments
Reliability degree

ABSTRACT

Decision analysis models often require the assessments of uncertain events elicited from informed experts to support the decision-making process. Expert opinions are often polled but their fusion is frequently beset by a number of difficulties pertaining to conflict and imperfection. Decision makers need, therefore, to reconcile inconsistencies by fusing the information provided by multiple sources of expertise. To reduce conflict and manage imperfection, expert information, represented by belief functions, need to be discounted proportionally to the degree they contribute to the conflict and its imperfection. The present study proposes a novel approach for determining the discounting operator of the information provided by a set of experts based on multiple criteria using the PROMETHEE II method. Expert judgments are then discounted and combined. Simple numerical examples and Monte Carlo simulations, including tests and comparative analysis with current approaches in the literature, are presented to illustrate the potential of the proposed approach.

1. Introduction

When making major decisions, decision makers turn to experts for advice as expert knowledge is often regarded as the best or only source of information. Although there are situations where one relies on advice from a single expert, in most cases advice can be solicited from multiple experts. Consulting the opinion of experts implies benefiting from their aptitudes, including knowledge in more or less related situations, theoretical background on the subject, and ability to establish meaningful analogies. Experts can have different degrees of expertise and are, therefore, likely to provide significantly different and heterogeneous opinions.

To cope with heterogeneity, most researchers combine all expert opinions [1]. Ouchi [2] suggests that there exist three different manners to fuse expert opinions: probabilistic risk analysis [3,4], fuzzy numbers theory [5], and belief function theory [6]. These three techniques require the assignment of probabilities or other numerical values by experts in order to model the uncertainty and imperfection of opinions. As pointed out by Sandri et al. [5], uncertainty models play a crucial role in the evaluation of expert opinions since none can affirm with absolute certainty his judgment or advice. In other words, judgments solicited from experts are frequently imprecise, incomplete, uncertain and, therefore, unreliable because of the inherently restricted precision of human assessments. In this context, unreliability is not equated with the full absence of reliability but is also taken to imply partial reliability (since the degree of reliability varies from one expert to another).

Pearl [7] affirms that belief functions are well-suited to represent expert judgments. In fact, belief function theory has commonly been used for modeling and fusing multi-expert judgments. Those judgments usually have some degree of bias, which is difficult, if not impossible, to quantify and remove. Several proposals have been advanced in the literature [8–18] to overcome the inadequacies associated with the conflicts and biases of expert opinions particularly through combination rules that allow the fusion of expert opinions. Considering the promising opportunities that this line of research might open with regards to the overcoming of biased and conflicting judgment problems, the present study was undertaken to develop a principled approach that allow for the discounting and combination of expert judgments (pieces of evidence). This work is particularly interested in the estimation of the discounting coefficient (reliability degree) associated with each expert judgment using the multi-criteria aggregation method PROMETHEE II. The reliability degrees estimated are then used to discount expert information, and the discounted information is eventually fused using the Dempster combination rule.

Several researchers [14,18–24] calculated the reliability degree attributed to each expert judgment using a single criterion. The assessment of expert information based on a single criterion is not, however, reliable enough since the mono-criterion approach is often insufficient to reflect reality. In fact, the conflict between expert judgments and the imperfection of expert information must

* Tel.: +216 98 571 500; fax: +216 74 460 744.
E-mail address: ahmed.frikha@isgis.rnu.tn

1566-2535/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.inffus.2013.04.010

be both taken into consideration during the calculation of reliability degrees. Accordingly, the present study opts for the use of the Multi-Criteria Decision Aid (MCDA) to estimate the reliability degrees of expert judgments.

Section 2 provides an inventory of the basic concepts of belief function theory. Section 3 is devoted to providing an overview of the major approaches of conflict management in this theory. Section 4 presents assessment criteria. The PROMETHEE II method is introduced in Section 5. Section 6 presents a new approach for reliability degree estimation based on the PROMETHEE II method and describes the process of discounting and combining expert judgments. Section 7 provides various numerical examples and Monte Carlo simulations, including tests, sensitivity examination, and comparative analysis with currently available methods which aim to illustrate the potential of the multi-criteria approach presented in this study. Finally, the last section contains a brief conclusion and avenues for further research.

2. The fundamentals of belief function theory

Belief function theory is initially introduced by Dempster [25], later formalized by Shafer [26] and axiomatically justified by Smets [27] in a transferable belief model. It is a general framework for reasoning with uncertain, incomplete and imprecise information. This theory is often reported to represent a promising alternative for information fusion and decision making using combination and decision rules, respectively.

2.1. Basic concepts

A belief function model is defined by a finite set Θ called *frame of discernment*. This set is viewed as a set of probable states to a given problem. The set including all possible subsets of Θ is termed the *power set* and is designated by 2^{Θ}.

A *basic probability assignment function* (BPA) is a mapping m: $2^{\Theta} \to [0,1]$. It assigns to every subset $A \subseteq \Theta$ a number $m(A)$, called the mass of A, which represents the degree of belief attributed exactly to A, and to no one of its subsets. This function must verify the following conditions: $m(\emptyset) = 0$, and $\Sigma\{m(A)/A \subseteq \Theta\} = 1$.

When $m(A) > 0$, A is termed *focal element* of m. In fact, the set of focal elements of m is designated as $\Im$ and the pair $(\Im, m)$ is named *body of evidence* (BOE).

For each BPA, we can associate a belief and plausibility functions. A *belief function* is a mapping Bel: $2^{\Theta} \to [0,1]$, defined as:

$$Bel(A) = \sum_{B \subseteq A} m(B) \ \forall\ A \subseteq \Theta \tag{1}$$

$Bel(A)$ measures the total belief completely attributed to $A \subseteq \Theta$.

A *plausibility function* is a mapping Pl: $2^{\Theta} \to [0,1]$, defined as:

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B) \quad \forall\ A \subseteq \Theta \tag{2}$$

$Pl(A)$ can be viewed as the maximum amount of belief that could be potentially given to A. In addition, it is possible to state that plausibility can be derived from belief:

$$Pl(A) = 1 - Bel(\overline{A}) \tag{3}$$

where $\overline{A}$ is the complement of A.

In belief function theory, total uncertainty (total ignorance) is expressed by $m(\Theta) = 1$ and $m(A) = 0$ for all $A \neq \Theta$. The associated belief function is defined by: $Bel(\Theta) = 1$ and $Bel(A) = 0$ for all $A \neq \Theta$, and is called *vacuous belief function*. Total certainty is expressed by $m(\{\theta_i\}) = 1$ for one particular element of Θ and $m(A) = 0$ for all $A \neq \theta_i$.

According to Smets' two-level view in transferable belief model (credal level where beliefs is maintained and represented by belief function, and pignistic level where beliefs are used to make decision and represented by probability function) [27], a BPA m must be transformed into *pignistic probability function BetP*. This transformation consists of equally distributing each mass $m(A)$ between the statements that compose $A \subseteq \Theta$. Formally, $BetP$ is defined as:

$$BetP(A) = \sum_{B \subseteq \Theta} m(B)(|A \cap B|/|B|) \quad \forall A \subseteq \Theta \tag{4}$$

where $|B|$ refers to the cardinality of a subset B. $BetP(A)$ can be viewed as the betting commitment to A and represents the total mass value that A can occur.

2.2. Discounting operation

When a BOE $i(\Im_i, m_i)$ is provided by unreliable experts, this unreliability is taken into account in belief function theory through the discounting operation. This is performed using the concept of *discounting operator* α_i associated to each BOE i. Firstly introduced by Shafer [26], this operator α_i quantifies the reliability of the BOE i. This operator varies between 0 and 1: the closer to 1, the greater the reliability is. The discounting operation is defined as follows:

$$\begin{cases} m_i^{\alpha_i}(A_j) = \alpha_i \cdot m_i(A_j) & \forall A_j \in 2^{\Theta} \setminus \{\Theta\} \\ m_i^{\alpha_i}(\Theta) = 1 - \alpha_i + \alpha_i \cdot m_i(\Theta) \end{cases} \tag{5}$$

where A_j refers to the focal element and $\alpha_i \in [0,1]$.

The discounting operation is based on the idea that each BPA mass is proportionally reduced, except for the mass of Θ which incorporates all the missing masses. If $\alpha_i = 1$, the BPA m_i is unchanged. However, if $\alpha_i = 0$, the result is a vacuous belief function.

2.3. Dempster combination rule

Combination is an operation that plays a crucial role in belief function theory. The BPAs induced by several distinct experts are combined to yield a global BPA that synthesizes the judgments of the different experts. The combination is performed using an aggregation rule whose application requires the independence condition for the experts to be combined. Let us denote by m_i and m_j two BPAs obtained from two distinct experts i and j in the same frame of discernment Θ. According to Dempster combination rule [26], we have:

$$m(A) = \frac{1}{1-K} \sum_{B \cap C = A} m_i(B) \times m_j(C) \quad \forall\ A \subseteq \Theta,$$
$$\text{where } K = \sum_{B \cap C = \emptyset} m_i(B) \times m_j(C) \tag{6}$$

The function m is called the orthogonal sum of m_i and m_j, and is denoted by $m = m_i \oplus m_j$. The coefficient K, which represents the mass attributed to the empty set, reflects the conflict between expert BOEs. The normalization factor $1 - K$ guaranties that no belief is associated to an empty set and the total belief is equal to one. The function m is not defined whenever $K = 1$. In this case, the two belief functions are totally contradicting each other. When $K = 0$, however, no conflict occurs between the two belief functions. The coefficient K can be viewed as the global conflict of combination.

The Dempster combination rule verifies some important properties (commutative and associative). It has, however, been criticized by several researchers [11,13,28,29] for its limited management of the conflict between different experts at the normalization stage and counterintuitive results when conflicting evidence is present.