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Estimation fusion algorithms in the presence of partially known cross-correlation of local estimation errors

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ABSTRACT

This paper addresses estimation fusion when the cross-correlation of local estimation errors is partially known. The statistical dependence of local estimation errors is first discussed, and then the concept of correlation coefficient is introduced to model the cross-correlation approximately. Two algorithms are proposed. One is based on min–max technique, which minimizes the maximal Mahalanobis distance between two fused estimates. The other one uses the prior distribution of the correlation coefficient and obtains a closed form of estimation fusion with the help of a series of matrix manipulations. Compared with some available algorithms in literature, simulation results demonstrate the effectiveness of the proposed approaches.

1. Problem formulation

Distributed fusion systems can be found in a large variety of applications such as aerospace, robots, environmental surveillance and so on [1,2]. In earlier researches, local estimation errors are assumed to be independent. However, in many applications, this assumption is not true. Many local estimation errors may be highly correlated in practical applications for the following reasons. One of the main causes is the common process noise that may get into both estimation errors. The other one is the correlated measurement noises, which happen when sensors operate in the same noisy environment, or when sensor measurements rely on the system state or the platform state. For instance, for a distributed fusion system with multiple navigation sensors installed in a motion vehicle, the measurement noises may be correlated since all the sensors are affected by the vehicle motion.

Another widely used assumption is that the covariance matrices of the full state are completely known. From a practical point of view, it is not possible. In many engineering applications such as map building or weather forecasting, the system is complicated enough with thousands of states [3]. Sometimes, it is also hard to obtain an accurate correlation between local estimation errors. This work focuses on providing a better estimate of the system state from noise-corrupted measurements when the cross-correlation is not completely known or the local estimation errors are not independent.

In order to simplify the analysis, we just consider a simple fusion model with two sensors and omit the time index. Let $\hat{\boldsymbol{x}}_i$ and $\boldsymbol{P}_i$ denote the local estimate and the corresponding covariance from the sensor i ($i = 1, 2$) respectively. The goal of estimation fusion is to obtain a formula to achieve a better fusion result $\{\hat{\boldsymbol{x}}, \boldsymbol{P}\}$, where $\hat{\boldsymbol{x}}$ and $\boldsymbol{P}$ denote the fused estimate and the resulting error covariance, respectively.

Influenced by the modeling error about the dynamic and measuring mechanisms, there exists the local estimation error between the local estimate $\hat{\boldsymbol{x}}_i$ $(i = 1, 2)$ and the real system state $\boldsymbol{x}$. Let $\tilde{\boldsymbol{x}}_i$ $(i = 1, 2)$ represent the local estimation error from sensor i ($i = 1, 2$) satisfying $\tilde{\boldsymbol{x}}_i = \boldsymbol{x} - \hat{\boldsymbol{x}}_i$. As stated above, local estimation errors from different local trackers might be highly correlated, resulting from the common process noise and the correlated measurement noises, etc. If the cross-correlation $\boldsymbol{P}_{ij} = E\left\{\tilde{\boldsymbol{x}}_i \tilde{\boldsymbol{x}}_j^T\right\}$ $(i, j = 1, 2;\ i \neq j)$ is exactly known, the BC (Bar-Shalom and Campo) fusion [1] in the sense of maximum likelihood (ML) can be employed in the following expressions.

$$\hat{\boldsymbol{x}}_{BC} = \hat{\boldsymbol{x}}_1 + (\boldsymbol{P}_1 - \boldsymbol{P}_{12})(\boldsymbol{P}_1 + \boldsymbol{P}_2 - \boldsymbol{P}_{12} - \boldsymbol{P}_{21})^{-1}(\hat{\boldsymbol{x}}_2 - \hat{\boldsymbol{x}}_1) \tag{1}$$

$$\boldsymbol{P}_{BC} = \boldsymbol{P}_1 - (\boldsymbol{P}_1 - \boldsymbol{P}_{12})(\boldsymbol{P}_1 + \boldsymbol{P}_2 - \boldsymbol{P}_{12} - \boldsymbol{P}_{21})^{-1}(\boldsymbol{P}_1 - \boldsymbol{P}_{21}) \tag{2}$$

However, the cross-correlation $\boldsymbol{P}_{ij}$ is not always available exactly. One of the main focuses in this paper is to explore the dependency structure and describe the cross-correlation in local estimation errors in an approximate manner. After that, based on

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the correlation model, two fusion schemes, denoted by Minmax and EBC (expected BC), are developed to integrate correlated local estimates in the context of the BC fusion [1].

This paper provides two algorithms for estimation fusion when the cross-correlation of local estimation errors are partially unknown. The basic assumption used is that the cross-correlation between local estimation errors can be represented by the correlation coefficient which is assumed to be in a known interval. The assumption made in this research is common since it is hard to determine the system model accurately, such as the transition model of system state, the measurement model, and the degree between measurement noises, etc. In this case, a fixed correlation coefficient is not enough to handle such a cross-correlation situation. In practical applications, the correlation coefficients can be determined in advance either by experts' experience or the Monte-Carlo simulation technique using a huge date set.

2. Literature review

Simple Convex Combination (SCC) approach [2] is the first method to implement estimation fusion by assuming that local estimation errors are statistically independent. Considering the effect of the cross-correlation, a recursive scheme was proposed to obtain the cross-covariance exactly in [4]. In 1986, Bar Shalom and Campo developed the BC algorithm [1] to integrate highly-correlated local estimates resulting from the common process noise. As indicated in [5], the BC algorithm is optimal only in the ML sense. In [6], the ML fusion was extended to the case when more than 2 sensors are employed, and it also pointed out that the fusion performance of [5] experienced a decrease with the increasing of the number of local sensors in comparison with the centralized estimation. In [7], the original BC formula [1] was generalized and the computational complexity was significantly reduced by an efficient iterative scheme.

For practical problems, it is impractical to compute the cross-correlation exactly. Recently, a robust filter called Covariance Intersection (CI) filter [3,8] was developed to integrate local estimates without any assumption about the cross-correlation of local estimation errors. It was proved that the CI algorithm yields a consistent estimate irrespective of the actual correlations.

In [9,10], some appropriate upper bounds of the covariance matrix were developed. Moreover, it was shown that the optimal weights in the CI algorithm can be obtained by minimizing the bounds. The CI algorithm achieves the consistency of the fused estimate by using a conservative fusion rule. However, this results in a decrease in the estimation performance and leads to a suboptimal solution. The largest ellipsoid algorithm was developed in [11], leading to a tighter estimate instead of overestimating the intersection region as done with the CI formula. An information theoretic explanation for the CI algorithm was given in [12], which demonstrates that the CI formula can be regarded as the log-linear combination of Gaussian densities. According to this relationship, the CI can also be used to fuse any two probability densities. In [13], an explanation based on the set theory about the CI approach was given. It is noted that optimizing a non-linear function is required by the CI approach, which is a significant drawback with respect to computational complexity. To solve this problem, some non-iterative and suboptimal algorithms were developed in [14,15] to compute the weights analytically. In [16], a relaxed Chebyshev center CI method based on the set theory and a fast CI (IT-FCI) method according to the information-theoretic metric were given. But the presented algorithms perform well only when the errors of local estimates have a negative correlation or a small positive correlation. In [17], a robust fusion problem with a norm-bounded uncertainty in cross-covariance was addressed. The proposed algorithm obtained an unbiased combination of local estimates by minimizing the worst-case mean squared error. In [18], the robust fusion problem was studied when auto-correlated and cross-correlated details are given.

From the above discussions, it is seen that the SCC is too optimistic, ignoring the inherent dependency of local estimation errors. On the contrary, the CI is too pessimistic which produces a bound on the estimation performance. The CI is obviously suboptimal in comparison with the algorithms which can exploit the information about the cross-correlation.

Although the cross-correlation is unknown exactly in general, some information about the dependency property might be possible to obtain for engineers, such as negative correlation, positive correlation and their corresponding correlation levels. If we can make a full use of the available information, it is possible to improve the fusion accuracy.

The most popular way assessing the overall association between two multivariate associations is CCA (canonical correlation analysis) [19]. In [20], an approximate correlation model was given, and the correlation coefficient is supposed to be a constant. In this paper, the supposition is extended to the case that the correlation coefficient may lie in an interval.

In view of the above fact, the goal of this paper is to develop fusion schemes which can utilize the partially-known prior of the cross-correlation. Our main contributions are threefold.

Firstly, the authors verified the validity and effectiveness of the correlation model in [20] by showing that the correlation coefficient in the correlation model behaves the same as the first correlation coefficient in CCA. Furthermore, instead of using fixed correlation coefficients, they can be uncertain, but they are within some intervals. Secondly, a min–max model is developed to minimize the maximal Mahalanobis distance between two fused estimates. Thirdly, a closed-form formula is derived from a series of matrix manipulations by assuming that the correlation coefficient follows a uniform distribution.

3. Estimation fusion with partially-known cross-correlation

3.1. Modeling the cross-correlation

Consider the accumulated vector generated by two n-dimensional local estimates $\hat{\hat{\boldsymbol{x}}} = \begin{pmatrix} \hat{\boldsymbol{x}}_1 \\ \hat{\boldsymbol{x}}_2 \end{pmatrix}$, its joint covariance can be described as

$$\Sigma = \begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_2 \end{bmatrix}$$

It is assumed that the cross-covariance $\boldsymbol{P}_{12}$ and $\boldsymbol{P}_{21}$ are unknown exactly. The objective of this subsection is to find a representation of the cross-covariance $\boldsymbol{P}_{12}$(possibly dependent on some correlation parameter) when given the covariance matrices $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$. It is noted that the objective here is different from general correlation analysis techniques (such as canonical correlation analysis (CCA), multivariate linear regression (MLA) [19] and so on), which aim to assess the correlation property when given a vast set of data points. However, what is needed here is a correlation model to evaluate the whole correlation property of local estimation errors, and only when the covariance $\boldsymbol{P}_i$ of random vector $\hat{\boldsymbol{x}}_i$ $(i = 1, 2)$ is available. When local estimates are scalar random variables, the correlation degree between them is well measured. However, the local estimates to be fused are random vectors generally for the fusion applications. In the cases of high dimension, it is difficult to interpret the cross-correlation in general. Here, we adopt the following model.