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Towards robust subspace recovery via sparsity-constrained latent low-rank representation

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ABSTRACT

Robust recovery of subspace structures from noisy data has received much attention in visual analysis recently. To achieve this goal, previous works have developed a number of low-rank based methods, among of which Low-Rank Representation (LRR) is a typical one. As a refined variant, Latent LRR constructs the dictionary using both observed and hidden data to relieve the insufficient sampling problem. However, they fail to consider the observation that each data point can be represented by only a small subset of atoms in a dictionary. Motivated by this, we present the *Sparse Latent Low-rank representation* (SLL) method, which explicitly imposes the sparsity constraint on Latent LRR to encourage a sparse representation. In this way, each data point can be represented by only selecting a few points from the same subspace. Its objective function is solved by the linearized Augmented Lagrangian Multiplier method. Favorable experimental results on subspace clustering, salient feature extraction and outlier detection have verified promising performances of our method.

1. Introduction

In recent years, it has emerged as a very important topic in visual analysis to explore the subspace structure of the noisy data, which has attracted much attention from both academia and industry [1–4]. The central goal is to capture the true subspace structure by eliminating noisy entries from original data, thus facilitating many real-world tasks, e.g., subspace clustering, classification and dimensionality reduction. To solve this problem, many promising approaches have been developed from various perspectives, such as robust principal component analysis (RPCA) [1], sparse representation (SR) [5,6] and low-rank representation (LRR) [7]. In this work, we focus on low-rank based methods as it has found wide applications in robust recovery analysis [2], subspace segmentation [8], matrix completion [9], etc.

Generally, the low-rank based representation methods can be cast into the popular topic, namely low-rank approximation [10–12]. LRR is one of the typical methods, which seeks the lowest-rank representation among all candidates representing the data points as the linear combination of the bases in a dictionary [7]. Mathematically, for a given observation matrix $\boldsymbol{X} \in \mathbb{R}^{m\times n}$ consisting of n data points, LRR aims to find the low-rank matrix $\boldsymbol{Z} \in \mathbb{R}^{n\times n}$ using

$$\min_{\boldsymbol{Z}} \quad \mathrm{rank}(\boldsymbol{Z}), \quad \text{s.t.} \quad \boldsymbol{X} = \boldsymbol{A}\boldsymbol{Z}, \tag{1}$$

where $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ is a dictionary. In LRR, the dictionary $\boldsymbol{A}$ is chosen as the observation matrix itself $\boldsymbol{X}$, which would depress the performance if the observations are insufficient and grossly corrupted. To address this issue, Latent LRR (LLRR) employs both observed and hidden data to construct the dictionary [13], which assumes all data points are sampled from the same collection of low-rank subspaces and the hidden effects can be approximately recovered by solving a nuclear norm minimization problem. However, an important yet heuristic fact is not respected by LLRR, i.e., each data point in the low-rank subspace can be represented by a linear combination within only a small subset of bases in the dictionary. In other words, the learnt low-rank representation should be simultaneously a sparse representation. For example, for a given data point $\boldsymbol{x}_i$, the vector $\boldsymbol{z}_i$ in the recovered term $\boldsymbol{A}\boldsymbol{z}_i$ is actually sparse.

Motivated by the progresses in sparse learning [5,14–16], we propose a novel low-rank based method named *Sparse Latent Low-rank representation* (SLL) by explicitly incorporating the sparsity constraint on the learnt low-rank matrix, thus obtaining a both lowest-rank and sparsest data representation. SLL is fundamentally based on LLRR, thus inheriting some advantages of LLRR, e.g., it can

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handle the hidden effects to compensate the insufficient sampling of the dictionary and is also able to extract the salient features from corrupted data. Furthermore, the sparsity constraint is superimposed on the objective function of SLL as a regularizer, leading to a more favorable structure of the data space and thus more robust to noise. To optimize the objective function, we adopt the variable splitting strategy [17] and the linearized Alternating Direction Method (ADM) [18], which shares the property of the Augmented Lagrange Multiplier (ALM) [19] method. Numerous experiments have shown promising results of the proposed method in comparison with others.

The remainder of this paper is structured as follows. We give a brief review on the related work in Section 2. The proposed Sparse Latent Low-rank representation (SLL) method is introduced in Section 3. Section 4 reports the experimental results with some analysis. In the end, we draw a conclusion in Section 5.

2. Related work

This section briefly reviews some closely related works to the proposed method. As a hot topic in pattern recognition and computer vision communities, robust recovery of subspace structures from corrupted data has gained much attention in the last decade. The popularity of this topic originates from the two aspects. On the one hand, the collected data points are often grossly corrupted with noises, outliers. On the other hand, it is strongly desirable to recover the clean data in several real-world applications [20,4,21–23], e.g., subspace segmentation, feature extraction, face recognition and outlier detection. To this end, there have emerged many approaches to handle this problem, such as robust principal component analysis (RPCA) [1,24,25], sparse representation (SR) [5,6,14] and low-rank representation (LRR) [7].

Among the above methods, both RPCA and LRR belong to the low-rank based methods while SR can be cast into the sparse coding paradigm [26]. RPCA as well as matrix completion is based on the hypothesis that data points are approximately drawn from a low-rank subspace, which does not cater to some situations when data points reside near several subspaces. To overcome this drawback, LRR assumes that samples are approximately drawn from a union of multiple low-rank subspaces. As an example of SR, sparse subspace clustering (SSC) concentrates on the scenario that samples are drawn from multiple low-dimensional linear or affine subspaces embedded in a high-dimensional data space [5]. RPCA recovers the low-rank component and the l_1-norm error term from the data matrix; SR attempts to learn a sparse representation of the corrupted data matrix; LRR seeks the lowest-rank representation as the linear combination of the bases in a given dictionary. SR may not capture the global structures of the data but LRR can better at discovering the global structures as it finds the lowest-rank representation of all data jointly [27]. This work focuses on the low-rank based methods.

The optimization of low-rank based methods appears to be very challenging due to the discrete nature of the rank function [28,29]. As indicated by matrix completion methods [30,31], the nuclear norm provides a good surrogate of the rank minimization problem [32], thus can be solved by convex optimization methods [33]. The objective function of LRR can be treated as a constrained convex program, which is often resolved by ADM limited to the case that the linear mappings in the constraints are identities. To deal with this limitation, a linearized ADM was proposed by linearizing the quadratic penalty term and adding a proximal term when solving the subproblems [18], thus making LRR be available for large-scale applications. To reduce the computational complexity of LRR, a recently proposed method, named fixed-rank representation (FRR) [34], incorporates the matrix factorization idea into low-rank representation [35]. A significant characteristic of LRR is to adopt the observed data matrix itself as the dictionary, which might depress the performance especially with insufficient or grossly corrupted observations. To address this shortcoming, Latent LRR (LLRR) construct the dictionary using both observed and hidden data, which are sampled from the same collection of low-rank subspaces [13].

Recent studies in sparse learning have shown that each data point is represented by a linear combination of only a few bases in the dictionary, which is often observed in the real-world applications [5,26,36,37]. However, the aforementioned low-rank based methods do not respect this key fact. Someone has considered both sparsity and low-rankness to construct the graph in semi-supervised learning [27], nevertheless, it also suffers from the insufficient sampling problem of the dictionary. Therefore, in this paper we consider explicitly imposing the sparsity constraint on the learnt low-rank representation by LLRR, thus capturing a both lowest-rank and sparsest data representation using both the observed and hidden data as the dictionary.

In addition, the core idea of this work can be extended in several aspects, such as semi-supervised learning [38], multi-view learning [39], weakly-supervised learning [40,41], discriminant learning [42,43]. In these directions, there have emerged a number of efficient computer vision methods.

3. Our approach

In this section, we describe our Sparse Latent Low-rank representation (SLL) method for robust recovery of subspace structures of the corrupted data. Besides, we adopt a linearized ADM method to optimize the objective function. We begin with the problem setting.

3.1. Problem setting

Given a set of data points $\mathbb{X} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\}$ drawn from a union of k independent subspaces $\{\mathcal{S}_i\}_{i=1}^{k}$ of low-rankness, a fraction of points are corrupted by noise or contaminated by outliers [7]. These points are stacked in columns of the matrix $\boldsymbol{X} \in \mathbb{R}^{m\times n}$, each of which can be reconstructed by a linear combination of the bases in the dictionary $\boldsymbol{A}$. While setting $\boldsymbol{A}$ as the observed data matrix $\boldsymbol{X}_o$, the ability of representing the underlying subspaces might be weakened, thus the unobserved hidden data matrix $\boldsymbol{X}_h$ is introduced as the other part of the dictionary as in LLRR [13], i.e., $\boldsymbol{X}_o = [\boldsymbol{X}_o, \boldsymbol{X}_h]\boldsymbol{Z}$, where $\boldsymbol{Z} = [\boldsymbol{z}_1, \ldots, \boldsymbol{z}_n]$ is the coefficient matrix with each component $\boldsymbol{z}_i$ as the new representation of $\boldsymbol{x}_i$. The main goal is to recover the subspace structure of the corrupted data $\boldsymbol{X}$.

3.2. Noiseless model

Consider the simple case that data are noiseless, we first review the latent low-rank representation, which aims to solve the following problem, i.e.

$$\begin{aligned} \min_{\boldsymbol{Z}} \quad & \operatorname{rank}(\boldsymbol{Z}), \\ \text{s.t.} \quad & \boldsymbol{X}_o = [\boldsymbol{X}_o, \boldsymbol{X}_h]\boldsymbol{Z}. \end{aligned} \tag{2}$$

Following [13], we show how to recover the hidden effects. Let $\boldsymbol{Z}^*_{o,h} = \left[\boldsymbol{Z}^*_{o|h}; \boldsymbol{Z}^*_{h|o}\right]$ be the optimal solution to $\min_{\boldsymbol{Z}} \operatorname{rank}(\boldsymbol{Z})$ with the same constraint, $\boldsymbol{Z}^*_{o|h}$ is a nontrivial block-diagonal matrix that can exactly reveal the true subspace membership when the sampling of the observed points is insufficient. To recover $\boldsymbol{Z}^*_{o|h}$ in the presence of $\boldsymbol{X}_o$ and $\boldsymbol{X}_h$, we assume both observed and hidden data are sampled from the same collection of low-rank subspaces. In general, it is impractical to exactly recover $\boldsymbol{Z}^*_{o|h}$ based on the