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Denoising by low-rank and sparse representations ☆

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ABSTRACT

Due to the ill-posed nature of image denoising problem, good image priors are of great importance for an effective restoration. Nonlocal self-similarity and sparsity are two popular and widely used image priors which have led to several state-of-the-art methods in natural image denoising. In this paper, we take advantage of these priors and propose a new denoising algorithm based on sparse and low-rank representation of image patches under a nonlocal framework. This framework consists of two complementary steps. In the first step, noise removal from groups of matched image patches is formulated as recovery of low-rank matrices from noisy data. This problem is then efficiently solved under asymptotic matrix reconstruction model based on recent results from random matrix theory which leads to a parameter-free optimal estimator. Nonlocal learned sparse representation is adopted in the second step to suppress artifacts introduced in the previous estimate. Experimental results, demonstrate the superior denoising performance of the proposed algorithm as compared with the state-of-the-art methods.

1. Introduction

This paper addresses the problem of image denoising in which the goal is to reconstruct the latent clean image $\mathbf{x}$ from its noise-degraded observation $\mathbf{y} = \mathbf{x} + \mathbf{w}$ where $\mathbf{w}$ is additive white Gaussian noise (AWGN) with zero mean and standard deviation σ. This problem is a classical yet active topic in low level image processing that serves as an important pre-processing step for many vision applications and provides a convenient test-bed over which various statistical image modeling methods can be assessed [1].

As image denoising is typically an ill-posed problem, the solution might not be unique. Therefore, natural image priors are widely used in order to regularize the possible solution spaces into the desired one. In fact, image priors are of utmost importance for an effective noise removal; hence various image priors have been developed [1]. A very strong prior for natural images is the nonlocal self-similarity (NSS) of small image patches within the image. The nonlocal means algorithm [2] is the first attempt to explicitly exploit nonlocal self-similarity for image denoising. This influential work has unleashed a flood of studies on nonlocal image restoration. Hence, many variants of nonlocal means [3–10] and several advanced nonlocal image denoising algorithms [11–20] have been developed.

Another well-known prior model is sparse representation which assumes that the clean signal can be well approximated by a linear combination of few basis elements – or atoms – from a set called a dictionary [21]. The idea of sparsity is traced back to the late 1980s when the sparsity of the wavelet coefficients was considered as an appropriate prior knowledge of natural images [21,22], leading to the famous shrinkage algorithm [23]. Evolution of this idea and development of overcomplete dictionaries introduced efficient and provably effective algorithms based on greedy pursuit [24,25] or convex optimization [26] to compute signal representations over arbitrary overcomplete dictionaries. Specializing the dictionary atoms in representing the intended signal and producing data adaptive dictionaries have motivated a wide range of investigations on dictionary learning during the last decade, resulting in several state-of-the-art dictionary learning algorithms such as K-SVD [27], online dictionary learning (ODL) [28], and recursive least squares dictionary learning (RLS-DL) [29].

By using the learned sparse representation models, promising results have been obtained in image and video denoising [22,30–32]. These methods produce patch-wise estimates and the final denoising results are calculated by aggregating the multiple estimates for pixels lying on the patch overlaps. Despite good denoising performance compared to point-wise estimators such as nonlocal means algorithm [2], these methods do not take into

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account the patch redundancy within image. It has been shown that joint use of sparsity and nonlocal self-similarity priors provides stronger image model, as exhibited in the state-of-the-art denoising methods BM3D [15] and LSSC [16]. The sparsifying dictionaries employed in these methods are kept fixed for all groups of similar image patches. However, sub-dictionaries adapted to each group could be used for better image modeling.

More recently, low-rank approximation for extracting low-dimensional structures in data has attracted attention in science and engineering resulting in an explosion of research in its theory and algorithms. Low-rank matrix approximation, for recovering of a low-rank matrix from its corrupted observation, appears in very many applications in various fields including computer vision, machine learning, signal processing, and bioinformatics. For instance, use of low-rank approximation can be found in applications such as face recognition [33], background modeling [34], medical image reconstruction [35,36], image alignment [37], video denoising [38], and image restoration [20,39] among others. As the generalization of sparse structures to correlated signals, low-rank approximation provides an effective approach toward modeling of nonlocal self-similarities in natural images.

In this paper, we develop a nonlocal image denoising approach in which two steps of low-rank approximation and sparse representation are employed. The first step of our algorithm is built upon the methodology of patch grouping and collaborative filtering where the proposed low-rank regularized collaborative filtering is applied. Indeed, noise removal from groups of matched image patches is formulated as low-rank matrix denoising. Based on recent results from random matrix theory, this problem is solved under asymptotic matrix reconstruction model leading to an optimal singular value shrinkage operator. In the second step, nonlocal learned sparse representation model is adopted to improve the shortcomings of the first step in flat image areas and to reduce artifacts around edges. This sparse model exploits the nonlocal redundancies to obtain a more accurate estimate of the original image.

In summary, our main contributions are as follows: (a) We exploit both sparsity and low-rank priors within a nonlocal denoising framework. (b) For the patch-based image denoising, we introduce a low-rank matrix estimator based on an optimal singular value shrinker, which does not require any threshold tuning. We prove that this shrinkage function can be applied to obtain optimal solution of weighted rank minimization problem with Frobenius norm data fidelity. (c) We apply nonlocal sparse representation model using a sparsifying overcomplete dictionary learned from the first-step estimate. The nonlocal redundancy is exploited to modify the initial sparse representations and produce a more accurate estimate of the original image.

The experimental results demonstrate that our proposed algorithm, called Sparse and Low-rank Representation based Denoising (SLRD), has superior performance compared with the state-of-the-art methods in both peak signal-to-noise ratio and visual quality.

The rest of the paper is organized as follows. In Section 2 we elaborate on the details of low-rank matrix recovery in presence of noise and its integration into a nonlocal denoising scheme. Nonlocal sparse model as second part of our algorithm is also described in this section. Experimental results, comparison with the state-of-the-art methods, and objective assessments are presented in Section 3. Finally, Section 4 concludes the paper.

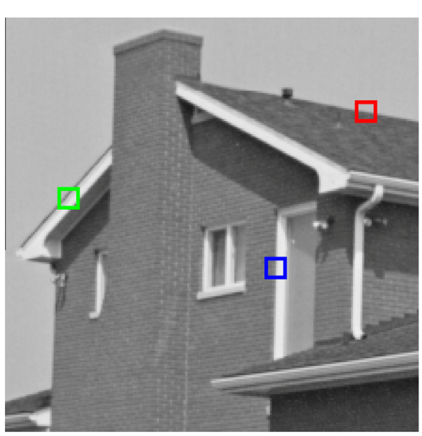
2. Proposed denoising scheme

Motivated by the great success of sparse representation and dictionary learning in various image restoration applications and considerable progress of low-rank approximation in recent years, we developed a new image denoising algorithm by considering both concepts of sparse representation and low-rank models. The denoising procedure is accomplished in two successive steps of the low-rank representation of nonlocal similarities and the sparse representation with respect to a learned overcomplete dictionary. The details of these steps are presented in the following subsections.

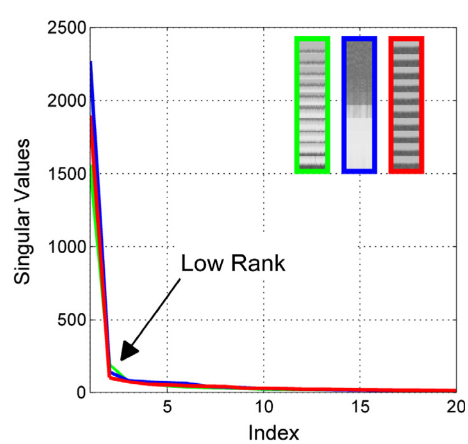
2.1. Step 1: Low-rank representation

In recent years there has been a surge of interest in data approximation by low-dimensional models such as sparsity, low-rank structures, and manifolds [40]. The basic idea of proposed Low-rank Representation based Denoising (LRD) approach is to approximate true noise-free image patches by low-rank modeling of image nonlocal similarities. In other words, the image patches are grouped by block matching, such that the patches in each group share similar underlying image structures. Thus, stack of these similar patches into a matrix, with vectorized image patches as its columns, forms a noisy version of an approximately low-rank matrix. This low-rank characteristic of image nonlocal similarities is demonstrated in Fig. 1. Based on this observation, recovery of original image patches can be cast as the problem of low-rank matrix recovery. This is done with the goal of estimating the latent low-rank matrix $\mathbf{X}$ from its noisy observation $\mathbf{Y}$:

$$\mathbf{Y} = \mathbf{X} + \sigma\mathbf{W} \tag{1}$$



(a)



(b)

Fig. 1. Low-rank characteristic of natural image blocks due to the nonlocal self-similarities. (a) "House" image, (b) singular values of three matrices formed from similar patches corresponding to the reference ones shown in (a). These matrices have been shown on top of singular values plot.