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A new similarity measure for non-local means filtering of MRI images

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ABSTRACT

In this paper, the application of non-local means (NLM) filtering on MRI images is investigated. An essential component of any NLM-based algorithm is its similarity measure used to compare pixel intensities. Unfortunately, virtually all existing similarity measures used to denoise MRI images have been derived under the assumption of additive white Gaussian noise contamination. Since this assumption is known to fail at low values of signal-to-noise ratio (SNR), alternative formulations of these measures which take into account the correct (Rician) statistics of the noise are required. Accordingly, the main contribution of the present work is to introduce a new similarity measure for NLM filtering of MRI images, which is derived under *bona fide* statistical assumptions and proves to posses important theoretical advantages over alternative formulations. The utility and viability of the proposed method is demonstrated through a series of numerical experiments using both *in silico* and *in vivo* MRI data.

1. Introduction

Magnetic resonance imaging (MRI) is considered to be one of the most advanced modalities of modern medical imaging, which excels in providing a spectrum of useful diagnostic contrasts [1]. The latter constitute an intensity-coded representation of the physical and physiological properties of biological tissues. Consequently, the precision with which a contrast is represented is of pivotal importance for the processes of tissue characterization. This fact establishes the principal value of post-processing techniques which aim at improving the signal-to-noise ratio (SNR) of diagnostic MR images, while preserving the integrity and consistency of their anatomical content.

Unfortunately, in virtually all realizations of MRI, attaining higher spatial resolution entails using longer acquisition times. Apart from being highly undesirable from the perspective of patients' comfort and compliance, longer acquisition times lead to motion-related artifacts, which are the main foe of cardiac and diffusion MRI [2–5]. On the other hand, decreasing the acquisition times is likely to result in a loss of spatial resolution as well as in an amplification of measurement noises. In addition to obscuring and masking diagnostically important details within MR scans, such noises also degrade the performance of computer-aided diagnosis, which necessitates the development and application of efficient and reliable tools of image denoising [6].

The current arsenal of image denoising methods used in MRI is immense, which makes their fair classification a difficult task. For this reason, we are going to limit our discussion to the following three groups of denoising techniques, which encapsulate a number of important approaches. Specifically, the first group encompasses variational methods, which are typically implemented through the numerical solution of certain partial differential equations (PDE) [7–11]. Thus, for example, [7] suggests an adaptation of the classical anisotropic diffusion filter of [12] for simultaneous noise reduction and enhancement of object boundaries in MRI. On the other hand, the denoising method of [8] is based on minimization of a different cost functional, whose associated gradient flow has the form of a fourth-order PDE. In [9], the information from body-coil and surface-coil images is incorporated into the reconstruction process in the form of data fidelity constraints. The work in [10] introduces a *maximum a posteriori* (MAP) technique using a Rician noise model in combination with spatial regularization. Finally, the work in [11] performs the denoising by combining the linear minimum mean square error (LMMSE) filtering [13] with anisotropic diffusion filtering.

A different group of denoising methods takes advantage of the sparsifying properties of certain linear transforms [14–23] (for a comprehensive review of such methods, the reader is also referred to [24]). Thus, for instance, the method of [18] is based on wavelet thresholding applied to squared-amplitude MR images, followed by "debiasing" of the approximation coefficients of the wavelet decomposition to account for their non-central chi-square distribution statistics. A different (robust) shrinkage scheme in the domain of a wavelet transform is also proposed in [21]. Using a different line of arguments, the wavelet denoising method of [20]

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is applied to complex-valued MR images. Finally, in [23], the MR images are enhanced by means of a wavelet-domain bilateral filter.

The third group of image denoising algorithms employs the concept of non-local means (NLM) filtering, which was originally proposed in [25–27], with its later improvements reported in [28,29]. As a general rule, NLM filters estimate the noise-free intensity of a given pixel as a weighted linear combination of the other (noisy) image intensities. In this case, the weights of the linear combination are determined based on a *similarity measure* (SM) between the neighbourhoods of the target and source pixels. As a result, the performance of an NLM filter is largely determined by the optimality of a chosen SM with respect to the properties of the original image as well as those of measurement noise. Thus, for example, under the conditions of additive white Gaussian (AWG) noise contamination, the NLM filters in [25,28] have been shown to outperform many variational and wavelet-based filters in terms of noise suppression and the quality of edge preservation.

Motivated by the success of NLM filtering in general imaging, the works in [30,31] have extended the Gaussian-mode NLM filters to MR imagery. A different approach is introduced in [32] which considers that the magnitude of MR images obey a Rician distribution [3]. This approach produces an unbiased estimate of the original image through combining a Gaussian-mode NLM filter with the operation of bias estimation and removal. A similar method has been reported in [33], where a Gaussian-mode NLM filter is applied to the squared magnitude of MR images. The same line of ideas have been further investigated and exploited in [34–36]. In the method reported in [37], the measured image intensities are first classified based on the similarity of their respective neighbourhoods, followed by estimating the image intensities by means of a maximum likelihood (ML) approach. A similar approach is derived in [38], where the similarity of image pixels is established based on a reference image, precomputed using the original NLM algorithm of [25–27]. Additionally, a variance stabilization approach for the Rice distribution has been proposed in the work of [39]. Subsequently, it has been demonstrated that the application of the variance stabilization transformation enables Gaussian-mode filters to achieve comparable denoising performance to filters specifically designed to account for the Rician statistics of the MRI noise. Finally, the approach in [40] applies the sparse 3D transform domain collaborative filtering approach of [28] to the case of MRI images.

In the majority of earlier methods for NLM-based denoising of MR images, the Rician nature of measurement noises is taken into account through either a rigorous ML analysis [37,38] or a properly designed "debiasing" procedure [32,33]. Common to these methods, however, is to compare pixel neighbourhoods using a similarity measure which is optimal for Gaussian noise contamination [41,42]. This fact suggests that a further improvement in the performance of NLM filtering could be obtained via adapting the similarity measure to the properties of Rician noise, which is inherent in MRI. Such an adaptation should be particularly useful in the case of relatively low values of SNR, whence the Gaussian (noise) model ceases to be an adequate approximation of the Rician one [3]. Accordingly, deriving such an SM constitutes the main objective of the present paper. The present work has been motivated by the results of [42,43], as well as by the more recent developments reported in [44,45]. Specifically, it has been pointed out that the approach for finding the similarity measure as presented in [42,43] should be avoided in the case of MR image denoising. Accordingly, we propose a new formulation of a similarity measure specifically designed for MRI noise, and demonstrate its usefulness and viability through a series of experiments using both *in silico* and *in vivo* MRI data.

Table 1 summarizes the main abbreviations used in the paper, whose remainder is organized as follows. Section 2 provides some necessary details on the image formation model of MR images and their noise statistics. Sections 3 and 4 describe some principal approaches to NLM filtering and point out their undesirable characteristics when applied to the MRI setting. A new SM and the closed-form expressions for its associated weights are derived in Section 5, while Section 6 details a method for applying the proposed weights to MR scans. Finally, Section 7 compares the performance of the proposed algorithm with that of some alternative methods using both *in silico* and *in vivo* MRI data, while the main results and conclusions of the paper are recapitulated in Section 8.

Table 1
List of Notations and Abbreviations.

Notations and Abbreviations	Meaning	Formula (if applicable)
NLM	Non-local means	-
NCCS	Non-central chi square	-
SM	Similarity measure	-
SNL	Similarity measure for NLM	-
SSM	Subtractive similarity measure	(14)
RSM	Rational similarity measure	(17)
$SSM_{s,t,k}$	SSM for *G*-images (NCCS statistic)	(16)
$RSM_{s,t,k}$	RSM for *M*-images (Rician statistic)	(19)
$C^G_{s,t,k}$	Correlative SSM for *G*-images (NCCS statistic)	(22)
$C^M_{s,t,k}$	Correlative RSM for *M*-images (Rician statistic)	(24)

2. Image formation model and noise statistics

MR images are acquired in the Fourier domain, followed by the processes of frequency demodulation and inverse transformation which result in complex-valued images, whose magnitude is subsequently displayed [4]. In this case, if the frequency-domain data is contaminated by zero-mean AWG noise, the complex amplitude M of the noisy observation $A\exp\{\imath\alpha\} + N$, with $N = N_r + \imath N_i$, is given by

$$M = \sqrt{(A\cos\alpha + N_r)^2 + (A\sin\alpha + N_i)^2}, \tag{1}$$

where A stands for the true image amplitude, while N_r and N_i are mutually independent AWG noises of standard deviation σ, and $\alpha \in [0, 2\pi)$ is an arbitrary phase shift. In this case, M can be shown to follow the Rician distribution model that is given by[1] [3,4]

$$p_{M|A}(m|a) = \begin{cases} \frac{m}{\sigma^2}\exp\left\{-\frac{a^2+m^2}{2\sigma^2}\right\} I_0\left(\frac{am}{\sigma^2}\right), & m \geqslant 0, \\ 0, & \text{otherwise}. \end{cases} \tag{2}$$

where I_0 denotes the zero-order modified Bessel function of the first kind. Fig. 1(a) depicts several typical shapes of $p_{M|A}(m|a)$ corresponding to a range of the values of A and $\sigma = 1$. As can be seen from the figure, for $A > 3\sigma$, the Rician probability density function closely resembles that of a Gaussian random variable [3]. However, for lower values of A, the density $p_{M|A}(m|a)$ becomes more asymmetric and protrudently heavy-tailed. Specifically, for $A = 0$, M follows a Rayleigh distribution model.

The Rician nature of M in (1) renders impractical a straightforward application of many filtering strategies. This is because of the highly-nonlinear relation between the expectation $\mathcal{E}\{M\}$ of M and A. Specifically, in the case of (2) one has

$$\mathcal{E}\{M\} = \sigma\sqrt{\pi/2}L_{1/2}(-A^2/2\sigma^2), \tag{3}$$

[1] Here and hereafter, we use the standard statistical formalism for denoting random variables and their associated realizations by capital letters and their lower-case counterparts, respectively.