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New results on connectivity in wireless network [☆,☆☆]

Min-Kuan Chang [a], Ting-Chen Chen [a], Che-An Shen [a], Wan-Jen Huang [b,*], Chih-Hung Kuo [c], Chia-Chen Kuo [d,1]

[a] No. 250, Kuo-Kuang Rd., South Dist., Taichung City 402, Taiwan
[b] No. 70, Lienhai Rd., Kaohsiung 80424, Taiwan
[c] No. 1, Daxue Rd., East Dist., Tainan City 701, Taiwan
[d] National Center for High-performance Computing, Taiwan

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A B S T R A C T

In this work, we discuss when two users are able to exchange multimedia content and when that exchange is always possible. Two thresholds are of interest, a threshold for percolation and a threshold for full connectivity. To derive these thresholds, when discussing the bond percolation, we consider not only four directions, upper, right, left, and lower, but also more directions like upper-right, lower-right, upper-left, lower-left. In addition, small structures are proposed to help obtain the necessary conditions for percolation and full connectivity. We find that if the probability of four directions being open is greater than 0.3118 or the transmission radius of every node is longer than $1.52d/k$, we can find a crossing path in our network and the percolation ensues. Furthermore, if that is greater than 0.3799 or the transmission radius of every node is longer than $1.579d/k$, a fully-connected network graph exists for sure.

1. Introduction

In recent years, percolation theorem is widely adopted to explain under what condition the connectivity in an observed phenomenon is possible. Percolation theorem can not only apply to the areas such as chemistry and physics, but also in the field of wireless network, which draws much attention recently [1]. With the help of percolation theorem, much research effort is devoted to discussing the conditions of establishing connection [2–5] and related issues such as the capacity of wireless network [6], node failure problems [7], broadcasting [8] and so on. In this work, we focus ourselves on the discussion of when two nodes in a wireless network can communicate with each other is possible in the same context. When two nodes in a wireless network can communicate with each other is of great importance and this is directly related to the success of a wireless network. Gilbert [2] first described the connectivity in a wireless network and proved that a wireless ad hoc network exists a phase-transitions behavior. However, in his model, the interference between nodes is not taken into account. Later, Dousse et. al. [4] discussed when the full connectivity is possible if the interference between nodes is present. In [4], the connectivity between two adjacent sub-squares means nodes in these two sub-squares can communicate one another and the conditions for percolation were proposed accordingly. The resulting graph describing how sub-squares are connected is unidirectional. That is, in that graph, if two sub-squares are connected, all nodes in these two sub-squares are connected. Later, Chang and his colleagues [5] further extended [4] to the situation where that graph is directional. In this situation, nodes in a sub-squares can communicate with all nodes in its adjacent sub-square but not the other way around. As indicated in Chang et al. [5], the transmission radius of a node specified in [4] is more than necessary to achieve full connectivity in a network. Under a certain condition, a smaller transmission radius of a node can still make the full connectivity in a network possible. Nevertheless, the common assumption of [2,4,5] is that the directions of connection are limited to the right, upper, left and lower directions. Considering the omnidirectional transmission, we relax the directions of connection to further include the upper-right, lower-right, upper-left and lower-left directions and a sub-square can connect to one or more adjacent sub-squares among its eight surrounding adjacent sub-squares.

In this paper we will use the same method as [4,5] to divide a wireless network into many sub-squares of size $d/k \times d/k$. Under

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* Corresponding author.
E-mail addresses: minkuanc@dragon.nchu.edu.tw (M.-K. Chang), ctc0724@gmail.com (T.-C. Chen), ann0929@hotmail.com.tw (C.-A. Shen), wjhuang@faculty.nsysu.edu.tw (W.-J. Huang), chkuo@ee.ncku.edu.tw (C.-H. Kuo), cckuo@narlabs.org.tw (C.-C. Kuo).
[1] No. 7, R&D 6th Rd., Hsinchu Science Park, Hsinchu City, Taiwan, 30076.

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the assumption of being populated [4,5], we propose a theorem to state when a wireless network is percolated based on the contrived small structures, which are first proposed in this work to help establish the conditions for percolation and full connectivity. The small structures proposed in this work are the most influential ways of connection to the adjacent sub-squares during the computation of the probability of a crossing path from the left to the right of the network. With the help of small structures, we are able to lower bound that probability, and based on the derived lower bound, the critical probability of having an open sub-edge and the corresponding critical transmission radius are found. We find that when the probability of having an open sub-edge is no less than 0.3118 and the transmission radius is no less than $1.5202d/k$, a wireless network percolates. Being percolated only tells us when two arbitrary might be able to exchange information but there is no guaranteed. Two arbitrary nodes can exchange their information for sure if and only if the network is fully connected. Thus, in this work, we establish the conditions to have a fully-connected network. When the probability of having an open sub-edge is no less than 0.37992 and the transmission radius is no less than $1.5792d/k$, all nodes in the network are connected. The extensive computer simulation is conducted and the results show the accuracy of the proposed theorem. This paper is organized as follows. First, we will review some prior arts in Section 2. In Section 3 we will introduce the system model with lattice construction. In Section 4 we will derive the conditions for percolation and use a simple method to find the critical value. The condition for full connectivity will be shown in Section 5. In Section 6, the simulation results are shown to demonstrate the accuracy of the proposed Theorem 1 followed by the concluding remark in Section 7.

2. Related works

Here we briefly review the results obtained in [2,4,5].

Gilbert [2] described the connectivity in a wireless network and proved that a wireless network exists a phase-transition behavior based on the continuum percolation theorem. Node i and node j with radius r are said to be connected if $|i-j| \leqslant r$, where $|\cdot|$ denotes the distance between two nodes. Under the definition of connectivity between two nodes, the phase transition can occur with the proper choice of r. In addition, there exists a critical value of r, say r^C. When r is no less than r^C, we will have a fully-connected wireless network. Otherwise, the wireless network is partitioned.

In [4], the authors used a way to map the continuous network graph to a discrete square lattice and the conditions for percolation in a network can be found by percolation theorem. In other words, we can use discrete networks to simulate continuous networks, e.g., bond percolation or site percolation [9]. In [4], authors divided a network into many finite smaller squares and further divided every finite small network into smaller sub-squares. The length of each sub-square is d/k. So every finite square is divided into k^2 sub-squares of size $d/k \times d/k$. There are four sub-edges in every sub-square. A particular sub-edge in a sub-square being open or not depends on whether the transmission range of a node in that sub-square could fully cover the adjacent sub-square which shares this sub-edge with that sub-square. In [4], when the transmission is no less than $\sqrt{5}d/k$, all four sub-edges of a sub-square are open. Under this condition, full connectivity ensues when each sub-square contains at least one node.

The mutually full coverage of two adjacent sub-squares is assumed in [4]. Under this assumption, all nodes in two adjacent sub-squares can connected to one another if the constraint on the transmission radius is met, and hence, the network graph of connectivity is deemed as being undirected. Hence, the symmetric routings, such as AODV [10], which assume that the same routing path is used for the source and the destination to exchange information, fit well in such an environment. However, this assumption is not always true. The source may use a route and the destination may take another one to send their one. This assumption underlies DSR [11]. This situation happens when not all nodes in two adjacent sub-squares can connect to one another. As a consequence, the network graph of connectivity is viewed as a directed graph. Chang and his colleagues [5] developed two theorems to answer when the network is percolated and when the network is fully connected in such a situation. Authors in Chang et al. [5] adopted similar approaches as Dousse et al. [4] and found that if there are at least four nodes in a sub-square, and the probability that a sub-square is closed is less than 0.5, then the network is percolated. In addition, under being populated, if the probability that a sub-edge of a sub-square is open is larger than 0.3822, then the network is fully connected almost surely. In this work, we further explore the same questions under the situation that a sub-square can connect to one or more of its eight neighboring sub-squares. This would be even more realistic than [5], which assumed that a sub-square can only connect to one of four direct adjacent sub-squares, which share sub-edges with this sub-square.

3. System model

The main idea is to divide the whole network into many squares, which are further divided into smaller sub-squares. By the analysis of the properties of sub-squares, we can find the desired results for the finite squares and then, induct the results for the entire network.

First, we construct a square lattice L over an infinite plane. Let the length of the square lattice L be d. We distribute nodes into the infinite plane following the Poisson point process in R^2 with density λ. Second, we divide the square lattice L into smaller squares called sub-squares. The length of every sub-square is d/k. So there are k^2 sub-squares in the square lattice L as shown in Fig. 1. In addition, the dual lattice of L, called L' is obtained by shifting L $d/2$ horizontally and vertically as shown in Fig. 1. We use dual lattice L' to support the lattice L in some situation. We have to introduce the following definitions to help develop the desired results:

Definition 1. A square is said to be populated if all its sub-squares contain at least one node.

Definition 2. The sub-edge is said to be open if the following conditions are fulfilled:

- The corresponding adjacent sub-square contains at least one node.
- The transmission coverage covers the corresponding adjacent sub-square completely.

The two definitions define the connectivity for one sub-square to the adjacent sub-squares [4]. By these definitions we can determine that whether a node can connect to another node in the adjacent sub-squares or not.

Definition 3. A random set of points $X \subset R^2$ is said to be Poisson process of density $\lambda > 0$ on the plane if it satisfies the conditions:

- For mutually disjoint domains of $R^2, D_1, \cdots, D_k$, the random variables $X(D_1), \cdots, X(D_k)$ are mutually independent, where $X(D)$ denote the random number of points of X inside domain D.