J. Vis. Commun. Image R. 32 (2015) 180–193

Contents lists available at ScienceDirect

J. Vis. Commun. Image R.

journal homepage: www.elsevier.com/locate/jvci

Nonconvex higher-order regularization based Rician noise removal with spatially adaptive parameters ☆

Myeongmin Kang [a], Myungjoo Kang [a], Miyoun Jung [b],*

[a] Department of Mathematical Sciences, Seoul National University, Seoul, Republic of Korea
[b] Department of Mathematics, Hankuk University of Foreign Studies, Yongin, Republic of Korea

ARTICLE INFO

Article history:
Received 31 March 2015
Accepted 8 August 2015
Available online 14 August 2015

Keywords:
Rician noise removal
Image denoising-deblurring
Variational minimization model
Nonconvex higher-order regularization
Nonconvex hybrid TV regularizer
Spatially adaptive parameter
Iteratively reweighted ℓ_1 algorithm
Alternating direction method of multipliers

ABSTRACT

In this article, we introduce a class of variational models for the restoration of images that are polluted by Rician noise and/or blurring. The novel energy functional consists of a convex fidelity term and a nonconvex higher-order regularization term. The regularization term enables us to efficiently denoise piecewise smooth images, by alleviating the staircasing effects that appear in total variation based models, and to preserve details and edges. Furthermore, we incorporate our nonconvex higher-order model with spatially adaptive regularization parameters; this further improves restoration results by sufficiently smoothing homogeneous regions while conserving edge parts. To handle the nonconvexity and nonsmoothness of our models, we adopt the iteratively reweighted ℓ_1 algorithm, and the alternating direction method of multipliers. This results in fast and efficient algorithms for solving our proposed models. Numerical experiments demonstrate the superiority of our models over the state-of-the-art methods, as well as the effectiveness of our algorithms.

1. Introduction

The last few decades have seen an advancement in the techniques for magnetic resonance imaging (MRI). However, noise corruption still frequently occurs during the image acquisition process. The noise occurs in the measured magnitude image, where the real and imaginary components are corrupted by zero-mean uncorrelated Gaussian noise with the same variance. Therefore, noise in the magnitude MRI image is modeled by a Rician distribution [1], whose probability density function is as follows:

$$P(r;\nu,\sigma) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2+\nu^2}{2\sigma^2}\right)I_0\left(\frac{r\nu}{\sigma^2}\right),$$

where I_0 is a modified Bessel function of the first kind with order zero. In this article, we consider the restoration of images corrupted by Rician noise and/or blurring.

Let $u:\Omega\to\mathbb{R}$ be an image defined on $\Omega\subset\mathbb{R}^2$, where Ω is a bounded open domain with a compact Lipschitz boundary. The format of the measured degraded image f under Rician noise and blurring is given by, [2],

$$f = \sqrt{(Au+\eta_1)^2+\eta_2^2},$$

where A is a known blurring operator, and η_1 and η_2 represent zero-mean uncorrelated Gaussian noise of standard deviation $\sigma > 0$.

In order to reconstruct a clean image u from the noisy data f, several models have been developed within a variational framework. First, Basu et al. [3] derived a log-likelihood term from the Rician distribution, and incorporated it into the anisotropic diffusion process of Perona–Malik [4], with the aim of denoising diffusion tensor MRI images. Moreover, Getreuer et al. [5] proposed a variational model that uses the negative log-likelihood term as a data fidelity term and total variation (TV) regularizer, for both Rician noise removal and deblurring,

$$\inf_{u\geqslant 0}\ \lambda E(f,Au) + \int_\Omega |\nabla u|\,dx, \tag{1}$$

where $E(f,Au) = \int_\Omega \left(\frac{f^2+(Au)^2}{2\sigma^2} - \log\left(\frac{f}{\sigma^2}\right) - \log I_0\left(\frac{f(Au)}{\sigma^2}\right)\right)dx$, $|\nabla u| = \sqrt{u_{x_1}^2+u_{x_2}^2}$, and $\lambda > 0$ is a parameter. The TV regularizer, introduced in [6], has been widely used in image processing, owing to its edge preserving property. Unfortunately, this minimization (1) is not convex, due to the fidelity term E, so the authors also proposed a convex approximation of the model (1). However, their convex model is complex, and its mathematical properties are difficult to derive. Recently, Chen and Zeng [2] proposed a new convex

☆ This paper has been recommended for acceptance by Prof. M.T. Sun.
* Corresponding author.
E-mail addresses: wjdjr1@snu.ac.kr (M. Kang), mkang@snu.ac.kr (M. Kang), mjung@hufs.ac.kr (M. Jung).

http://dx.doi.org/10.1016/j.jvcir.2015.08.006
1047-3203/

variational model, by inserting a quadratic penalty term into the model (1), which is given by

$$\inf_u \lambda\left(E(f,Au) + \frac{1}{\sigma}\int_\Omega (\sqrt{Au} - \sqrt{f})^2 dx\right) + \int_\Omega |\nabla u|\, dx, \qquad (2)$$

where $0 \leqslant u \leqslant 255$, and λ is constant parameter. This model is strictly convex with $\sigma \geqslant \sigma_0 := \sqrt{\frac{255}{3902}\sup_\Omega f}$ when A is the identity operator, and it is convex with $\sigma \geqslant \sigma_0$ when A is a blurring operator. With $\sigma \geqslant \sigma_0$, the authors proved the existence and uniqueness of solutions for problem (2), and numerical results demonstrated that the model (2) performs better than the model (1) for restoring images corrupted by Rician noise. From now on, we will call this convex model the "TV model."

On the other hand, Liu et al. [7] proposed a nonconvex total variation model for Rician noise removal, incorporated with a spatially adaptive regularization parameter, as follows:

$$\inf_u \int_\Omega (K \otimes \lambda)\left(\frac{1}{2\sigma^2}u^2 - \log I_0\left(\frac{fu}{\sigma^2}\right)\right) dx + \int_\Omega |\nabla u|^\gamma dx,$$

where K is a symmetric Gaussian kernel satisfying $K(x) = K(-x)$ and $\int K(x)\,dx = 1$, $\otimes$ denotes a convolution operator, and $\gamma \in (0,1)$. Here, the parameter $\lambda : \Omega \to \mathbb{R}$ is defined as a spatially varying function, depending on the pixel coordinate $x = (x_1, x_2)$. Despite the nonconvexity, this model takes advantage of local image features, and has achieved remarkable denoising results.

A variational model for image restoration usually consists of a data fidelity term and a regularization term. The regularization term encourages a smooth output image and eliminates noise in the presence of noise. Various convex regularizers have been proposed, such as TV [6] and higher-order regularizers [8–12]. Higher-order regularizers have been proposed in order to overcome the staircasing effects of TV regularizer, and so to effectively denoise piecewise smooth images. On the other hand, many studies [13–18] have demonstrated that nonconvex regularizers are superior to convex ones, due to their edge preserving property. In particular, the authors in [16] illustrated that the quality of TV denoising results can be improved by replacing the ℓ_1 norm of gradient by the nonconvex ℓ_q norm, for $0.5 < q < 0.8$. Nikolova et al. [17] also showed that various types of nonconvex regularizers are preferable to convex ones for the preservation of discontinuities in an image. Moreover, Oh et al. [18] recently proposed a nonconvex higher-order regularizer, and demonstrated its superiority over convex higher-order regularizers as well as nonconvex TV-based regularizers.

In the last decade, many efficient algorithms for minimizing nonconvex optimization problems have been proposed, with or without convergence analysis. For examples, gradient descent based methods [19,20], the half-quadratic algorithm [21,22], and the graduated nonconvexity continuation method [17] are the classical approaches for nonconvex optimization. Recently, Candés et al. [23] proposed the iteratively reweighted algorithm, for solving compressive sensing problems that involve a nonconvex log function instead of the ℓ_1-norm. Furthermore, Ochs et al. [24] extended this to the iteratively convex majorization–minimization method, for solving nonsmooth nonconvex optimization problems. Furthermore, they provided various versions of iteratively reweighted algorithms, with convergence analysis under certain conditions.

In general, the regularization parameter λ in a variational model controls the smoothness of the restored image. That is, small λ leads to oversmoothing of small features, such as edges and details, while large λ results in leftover noise in homogeneous regions. Therefore, in order to balance the quality and efficiency of image denoising, the spatially adaptive regularization parameter (SARP) approach has been employed in many works [7,25–29]. Recently, the SARP approach was utilized for Rician noise removal in [7], based on ideas from [26]. Their optimization algorithm is based on the gradient descent method, and regularization parameters are updated at each iteration, which results in a slow convergence to reach a solution. In our work, we propose a novel strategy for the automated selection of spatially adaptive parameters, based on the local expected value estimator, like [27,29]. In addition, we present a fast and efficient optimization algorithm for our model integrated with SARP.

In this paper, we introduce a class of variational minimization models for the restoration of images that are corrupted by Rician noise and/or blurring. The models consist of a convex fidelity term and a nonconvex higher-order regularization term, which enables us to effectively denoise piecewise smooth images while also preserving edges. Furthermore, we propose an automated adjustment strategy for the spatially adaptive parameter, based on an idea in [27,29]. This further improves the quality of restored images, by preserving fine scales while denoising homogeneous regions. In order to solve nonconvex nonsmooth problems, we utilize the iteratively reweighted algorithm [24] and the alternating direction method of multipliers. This results in fast and efficient algorithms. Numerical experiments demonstrate the superiority of our proposed models over existing methods, as well as the efficiency of our proposed algorithms.

The outline of the rest of this paper is as follows. In Section 2, we recall various regularizers proposed in prior works, and the iterative reweighted ℓ_1 algorithm in [24]. In Section 3, we propose a variational minimization model that involves a nonconvex higher-order regularization term. In Section 3.1, an optimization algorithm for solving the proposed model is illustrated. In addition, in Section 4, we combine our proposed model with SARP, and we describe an automated selection strategy for the SARP. In Section 5, numerical experiments are provided for our proposed models, along with comparisons with state-of-the-art methods. Finally, in Section 6, we summarize our work and provide some comments about future work.

2. Preliminaries

2.1. Convex and nonconvex regularizing functionals

First, Rudin et al. [6] introduced the TV regularizer for an image denoising model, as

$$\min_u \frac{\lambda}{2}\int_\Omega (u-f)^2 dx + \int_\Omega |\nabla u|\, dx,$$

where $\lambda > 0$ is a tuning parameter. This TV model can remove noise while also preserving edges well, but it tends to create piecewise-constant images, even in regions with smooth transitions of intensity values in the original image. It is often called staircasing effects.

In order to alleviate such artifacts, a higher-order version of TV was proposed in [9,11], which uses second-order derivative information. In particular, Lysaker et al. [11] proposed the following image denoising model:

$$\min_u \frac{\lambda}{2}\int_\Omega (u-f)^2 dx + \int_\Omega |\nabla^2 u|\, dx,$$

where $\nabla^2 u = \sqrt{u^2_{x_1x_1} + u^2_{x_1x_2} + u^2_{x_2x_1} + u^2_{x_2x_2}}$. This model reduces the staircase effects, and performs efficiently for the denoising of piecewise smooth images. However, this higher order TV results in less well preserved edges than TV in practice.