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Split Bregmanized anisotropic total variation model for image deblurring ☆

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ABSTRACT

In this paper, an effective image deblurring model is proposed to preserve sharp image edges by suppressing the stair-casing arising in the total variation (TV) based method by using the anisotropic total variation. To solve the difficult L1 norm problems, the split Bregman iteration is employed. Several synthetic degraded images are used for experiments. Comparison results are also made with total variation and nonlocal total variation based method. Experimental results show that the proposed method not only is robust to noise and different blur kernels, but also performs well on blurring images with more detailed textures, and the stair-casing effect is well suppressed.

1. Introduction

It often happens that an acquired image suffers from the blurring due to atmospheric turbulence, an out of focus camera, or relative motion between the camera and the object. Noise may be introduced into the image because of measurement errors, quantization and imperfection in the recording and transmission medium, and digitization, etc.

A blurred noisy image is often considered as the convolution of a clear image with blur kernel plus the additive noise:

$$f = Au + n \tag{1.1}$$

where f is the degraded image, u is the clean image to be estimated from f, n is the additive white Gaussian noise, and A is a convolution operator. Solving (1.1) is ill-posed due to the large condition number of A. Any small perturbation on the degraded image f may lead to the solution $A^{-1}f$ to be very far away from the true image u [1,2]. Many different approaches have been proposed for the deblurring problem.

One of the main methods is to use the regularization based methods, which minimize some cost functionals to find the solution. Tikohonov regularization is a simplest one, which minimizes an energy containing a data fidelity term with a L2 norm regularization term. When A is a convolution operator, the problem can be solved in the Fourier domain, this is the so-called Wiener filter [3]. However, the edges of recovered image are often smoothed. Later, a total variation (TV) based regularization was proposed in [4] to overcome this problem.

$$\text{TV/ROF model}: \min_u \|u\|_{BV} + \frac{\mu}{2}\|Au - f\|_2^2 \tag{1.2}$$

where $\|u\|_{BV} = \int_\Omega |\nabla u| dxdy$ is the total variation of u. TV has been widely used in image processing because of its advantage on preserving edges. However, it is well-know that TV yields unwanted stair-casing [5–7], which would cause that the restored image would lose some texture and corners.

Parallel to the TV method, the anisotropic diffusion method can also be applied to enhance the image edges [8].

$$\partial_t u + div\left(\frac{\nabla u}{1 + c|\nabla u|^2}\right) = 0$$

This method stops the diffusion at edges, which can be described by steep gradients, while treating flat regions as in the uniform model. However, anisotropic diffusion method has the drawbacks that the solutions are in fact smooth and it has the same problem of speckles as the isotropic diffusion [9,10], because the slow oscillations cannot be penalized sufficiently by the quadratic regularization term.

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To use the advantages and modify the drawbacks of TV and anisotropic diffusion methods, a new anisotropic regularization and diffusion method has been implemented for the restoration of polygonal shapes with sharp edges and corners. This new anisotropic regularization is the so-called anisotropic total variation (ATV) which is developed by Esedoglu and Osher [11]. For more detail therein, see [12]. The anisotropic total variation is defined for all $u \in BV(R^2)$ as

$$\mathrm{ATV}(u) = \int_\Omega (|\nabla_x u| + |\nabla_y u|)dxdy \tag{1.3}$$

Model (1.3) has been successfully applied to image denoising [13–15,27], inpainting [17] and 2D bar codes restoration [16]. However, up to now, few of works have been done in image deblurring by using ATV. In this paper, an image deblurring method based on ATV is proposed. In addition, a unified ATV restoration model for simultaneous image denoising and deblurring is established. Although there exists many other methods (e.g., anisotropic local likelihood approximations, nonlocal total variation and 3D Transform-Domain Collaborative Filtering) which can obtain the high quality image with abundant structure and features [31–36]. Here, we only focus on the anisotropic total variation.

The TV functions (e.g., ROF model) have been shown to be computationally difficult to solve by conventional methods due to their nonlinearity and non-differentiability. Although many authors have proposed improved schemes, such as time marching scheme, primal–dual variable strategy, fixed-pointed iteration algorithm (for reviews see [23,30]), the optimization problem of (1.2) is still difficult to solve due to the existence of L1 norm based term. In [18], Bregman iteration method was introduced for the nondifferentiable TV function, and has been proved to be spectacularly successful for L1 norm minimization problems [19]. Later, a linearized Bregman iteration was proposed to improve the function of the Bregman iteration [20] (detailed theory see [21,22]). Recently, split Bregman iterations were developed in [23], which extended the application of the Bregman iteration and the linearized Bregman iteration to more general L1 norm minimization problems [24,25].

In this paper, we proposed a split version of the anisotropic total variation for simultaneous image denoising and deblurring, which is very efficient in persevering edges and corners of the image with fewer stair-casing effect arising in ROF model [4]. Instead of solving the Lagrange equations directly, we introduce a new unconstrained problem by applying operator splitting and penalty techniques to take replace of the original minimizing issue.

The rest of the paper is organized as follows. In Section 2, a brief introduction to split Bregman iteration is presented. In Section 3, an unified ATV model based on split Bregman iteration is derived. In Section 4, the applications of the model in Section 3 are extended to image denoising and image deblurring based on different linear operator A. Numerical results are illustrated in Section 5.

2. Split Bregman iteration

In this section, we give a brief introduction about how the Bregman framework derive from Bregman iteration to solve the L1 norm optimization problem, for detailed theory about Bregman iteration see [18].

2.1. Bregman distance and Bregman iteration

Consider two convex energy functions, $E(u)$ and $H(u)$. The minimization problem to be considered is

$$\min_u E(u) + H(u) \tag{2.1}$$

The Bregman iteration comes from the concept of 'Bregman distance'.

$$D_E^p(u, u^k) = E(u) - E(u^k) - (p, u - u^k) \tag{2.2}$$

Then the Bregman iteration for (2.1) is

$$u^{k+1} = \min_u D_E^p(u, u^k) + H(u) \tag{2.3}$$

$$u^{k+1} = \min_u E(u) - E(u^k) - (p, u - u^k) + H(u) \tag{2.4}$$

In order that (2.4) is well defined for $k+1$, it must hold: $p^{k+1} = p^k - \nabla H(u^{k+1})$. Then

$$p^{k+1} = p^k - \nabla H(u^{k+1}) \tag{2.5}$$

When $H(u) = \frac{\lambda}{2}\|Au - f\|_2^2$ and A is linear, the Bregman iteration of (2.4) and (2.5) is equivalent to the following simplified version:

$$u^{k+1} = \min_u E(u) + \frac{\lambda}{2}\|Au - f + p^k\|_2^2 \tag{2.6}$$

$$p^{k+1} = p^k + Au^{k+1} - f \tag{2.7}$$

2.2. Split Bregman iteration

Consider the general L1 problem, assume $E(u) = |d|$ and $d = \Phi(u)$, then (2.1) can be modified by

$$\min_{u,d} |d| + H(u) \quad \text{such that } d = \Phi(u) \tag{2.8}$$

Here, assume $H(u)$ and $|\Phi(u)|$ to be convex function, also assume $\Phi(u)$ to be differentiable. To solve this problem, first convert it into an unconstrained problem:

$$\min_{u,d} |d| + H(u) + \frac{\mu}{2}\|d - \Phi(u)\|_2^2 \tag{2.9}$$

If we let $W(u) = |d| + H(u)$, (2.9) becomes the Bregman iteration, then use (2.6) and (2.7) we get

$$(u^{k+1}, d^{k+1}) = \min_{u,d} |d| + H(u) + \frac{\mu}{2}\|d - \Phi(u) - b^k\|_2^2 \tag{2.10}$$

$$b^{k+1} = b^k + (\Phi(u^{k+1}) - d^{k+1}) \tag{2.11}$$

(2.10) and (2.11) are the two iteration models of the split Bregman iteration.

In order to implement (3.7), we must minimize the subproblems of u and d respectively with the following steps:

$$\text{Step 1}: \; u^{k+1} = \min_u H(u) + \frac{\mu}{2}\|d^k - \Phi(u) - b^k\|_2^2 \tag{2.12}$$

$$\text{Step 2}: \; d^{k+1} = \min_d |d| + \frac{\mu}{2}\|d - \Phi(u^{k+1}) - b^k\|_2^2 \tag{2.13}$$

To compute d, Osher uses soft shrinkage operators as:

$$d_j^{k+1} = shrink(\Phi(u)_j + b_j^k, 1/\mu) \tag{2.14}$$

where

$$shrink(x, \gamma) = \frac{x}{|x|} * \max(|x| - \gamma, 0)$$

The split Bregman iteration has several same good properties as Bregman iteration which have been discussed and proved in [18,19]. Also, it can converge very quickly when applied to the L1 regularization problem and avoid the problem of numerical instabilities that occur as $\mu \to \infty$ when using continuation methods.