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Image denoising via local and nonlocal circulant similarity ☆

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ABSTRACT

A patch based image denoising method is developed in this paper by introducing a new type of image self-similarity. This self-similarity is obtained by cyclic shift, which is called "circulant similarity". Given a corrupted image patch, it can be estimated by incorporating circulant similarity into a weighted averaging filter. By choosing an appropriate kernel as weight function, the patch filter is implemented by circular convolution, and can be efficiently solved using fast Fourier transform. In addition, the circulant similarity can be enhanced by using nonlocal modeling. We stack the similar image patches into 3D groups, and propose a denoising scheme based on group estimation across the patches. Numerical experiments demonstrate that the proposed method with local circulant similarity outperforms much its local filtering based counterparts, and the proposed method with nonlocal circulant similarity shows very competitive performance with state-of-the-art denoising method, especially on images corrupted by strong noise.

1. Introduction

The goal of image denoising is to faithfully reconstruct an image $\mathbf{x}$ from its noise corrupted observation $\mathbf{y} = \mathbf{x} + \mathbf{v}$, where $\mathbf{v}$ is commonly assumed to be additive white Gaussian noise with zero mean and standard deviation σ in literature. Image denoising is a fundamental problem and an indispensable process for many image processing and low level vision applications, while it provides an integral platform for investigating the statistical modeling of natural images.

Earlier image denoising methods include Gaussian filtering [1] and anisotropic diffusion [2,3]. The total variation (TV) minimization [4] is a classical nonlinear anisotropic diffusion model by minimizing an energy functional. Since natural images are highly redundant, it is expected that they can be de-correlated and more compactly represented in some transformed space. Wavelet transform [5] is such a tool and it decomposes the input image into multiple scales, and at each scale some statistical modeling method can be applied to suppress noise [6]. Principle component analysis has also been used to learn locally adaptive transforms from the image, and it has shown very good performance in local structure preservation [7,8].

Since denoising is a typical ill-posed problem, the using of natural image priors to regularize the solution is critical to the success of a denoising algorithm. It is widely accepted that natural image gradients exhibit heavy-tailed distributions, which can be fitted by Laplacian or hyper-Laplacian models [9,10]. The TV based methods [4] actually assume Laplacian distributions of image gradients. Many statistical models of wavelet coefficients have been proposed, such as generalized Gaussian [11] and Gaussian scale mixture [12] models, etc. In [13], the authors used a mixture of bivariate Laplacian probability density functions for the clean data in the transformed domain. Later, Yu et al. [14] used multivariate Gaussian priors of image patches for solving image inverse problems. Based on the fact that natural images can be sparsely represented in some domain, the sparse representation (SR) techniques [15,16] perform nonlinear sparse coding to effectively exploit the sparsity of natural images. SR and its variants have achieved impressive denoising results [17,18]. The dictionary employed in SR is crucial to the final denoising results. Owe to the seminal work of KSVD [19], learning dictionaries from the natural images has become a hot topic in image processing and computer vision [20–22].

How to measure and exploit the similarity between image pixels is a critical issue in image denoising. Instead of using only the spatial location to compute the similarity of neighboring pixels, the well-known bilateral filtering method [23] computes both the spatial and intensity similarities for pixel averaging. The seminal work of nonlocal means (NLM) [24] brings image denoising from the era of local similarity to the era of nonlocal self-similarity

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(NSS); that is, the similar pixels to a given pixel can be spatially far from it. Inspired by the success of NLM, Dabov et al. [25] constructed 3D cubes of nonlocal similar patches and conducted collaborative filtering in the sparse 3D transform domain. The so-called BM3D algorithm has been a benchmark in image denoising. Mairal et al. [18] exploited the NSS via $l_{p,q}$-norm simultaneous sparse coding. Zontak and Irani [26] proposed an internal parametric prior to evaluate the nonlocal patch recurrence, and they recently found that patch recurrence also holds across scales [27]. The nonlocally centralized sparse representation model developed by Dong et al. [17] shows very powerful denoising performance. By assuming that the matrix of nonlocal similar patches is of low rank, the low-rank minimization based methods [28,29] have also achieved interesting denoising results.

In this work, we propose a new image denoising method by exploiting the image local and nonlocal circulant similarity. We observe that many small patches in natural images will show similar patterns after a certain cyclic shift. This local circulant similarity can be used as a prior for image denoising. By using circulant shift, the proposed model becomes very simple and relies solely on the kernel circulant matrix. Moreover, the computational cost can be much reduced by fast Fourier transform (FFT). In addition, circulant similarity can be enhanced by using nonlocal techniques. We stack the similar image patches into 3D groups, and propose a group estimation method by performing circulant similarity along the three dimensions. The proposed method with local circulant similarity shows clear advantages over its local filtering based counterparts, while the proposed method with nonlocal circulant similarity shows very competitive performance with the state-of-the-art nonlocal based denoising method, especially on high noise levels.

The rest of the paper is organized as follows. In Section 2 we introduce local circulant similarity and propose a 2D patch estimation method by using local circulant similarity. In Section 3 we propose a 3D group estimation method by using nonlocal circulant similarity for image denoising. In Section 4 we present experimental results to demonstrate the advantages of the proposed model.

2. Denoising by local circulant similarity

2.1. Local circulant similarity

A natural image usually contains many local similar features and nonlocal repetitive patterns, such as textures of background and edges of object. Particularly, image patches often have local circulant similarity. Circulant similarity is a special type of self-similarity, which refers to that a patch shows similar pattern after a certain cyclic shift of it. Such circulant similarity can be measured by Gaussian kernel function κ, which will be shown in detail later. Denote by m_k the maximum similarity between the original patch and its cyclic shifted versions. We uniformly extract 373,000 image patches (size: 9×9) from 12 popularly test images to show local circulant similarity exists widely in natural images. As shown in Fig. 1, these patches are divided into 3 classes according to their degrees of circulant similarity: strong circulant similarity (SCS, $m_k \geqslant 0.8$), weak circulant similarity (WCS, $0.8 > m_k \geqslant 0.4$), and no circulant similarity (NCS, $m_k < 0.4$). The proportions of SCS, WCS and NCS are 40.6%, 35.9%, and 23.6%, respectively. It can be observed that there exist a lot of patches with strong or weak circulant similarities in natural images. The SCS and WCS could be used as image priors for image denoising, especially in the presence of severe noises. Based on this insight, we present a novel patch estimation method for image denoising.

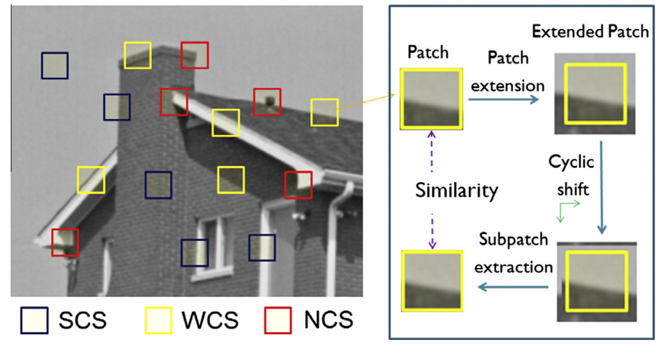

Fig. 1. Illustration of local circulant similarity.

2.2. Patch estimation

Denote by $\mathbf{u}$ the observed noisy image patch, $\mathbf{u} = [u_0, \ldots, u_{n-1}]^\top$. Here, we stick to definitions for 1D signals for simplicity of the notations. The extension to 2D and 3D is straightforward. We define a shift operator $T^l : T^l(v_0, \ldots, v_{n-l-1}, v_{n-l}, \ldots, v_{n-1}) = (v_{n-l}, \ldots, v_{n-1}, v_0, \ldots, v_{n-l-1})$, where l is the number of shifted elements to the right, and we have $T^0 = T, T^{n+l} = T^l$. Let u_i be the noisy pixel value at position i. For arbitrary pixel $u_j \in \mathbf{u}$, we build its neighborhoods $\mathbf{p}_j$ by patch cyclic shift, $\mathbf{p}_j = T^j\mathbf{u}$. The estimated pixel $\hat{u}_i$ by weighting u_j can be described as:

$$\hat{u}_i = \alpha_i \sum_{j=0}^{n-1} \kappa(\mathbf{p}_i, \mathbf{p}_j) u_j \tag{1}$$

where the weight function $\kappa(\cdot)$ measures the similarity between the patches $\mathbf{p}_i$ and $\mathbf{p}_j$, and α_i acts as a normalizing constant $\alpha_i = 1/\sum_j \kappa(\mathbf{p}_i, \mathbf{p}_j)$. For simplicity, we omit the normalizing factors α_i from the equation. Thus the estimate $\hat{u}_i$ can be reformulated as

$$\hat{u}_i = \sum_{j=0}^{n-1} u_j \kappa(T^i\mathbf{u}, T^j\mathbf{u}) = \sum_{j=0}^{n-1} u_j K_{i,j} \tag{2}$$

By stacking all the pixels together, the patch estimate can be represented in a matrix–vector multiplication form

$$\begin{bmatrix} \hat{u}_0 \\ \vdots \\ \hat{u}_{n-1} \end{bmatrix} = \begin{bmatrix} K_{0,0} & \cdots & K_{0,n-1} \\ \vdots & \ddots & \vdots \\ K_{n-1,0} & \cdots & K_{n-1,n-1} \end{bmatrix} \begin{bmatrix} u_0 \\ \vdots \\ u_{n-1} \end{bmatrix}$$

The above equation can also be written as

$$\hat{\mathbf{u}} = K\mathbf{u} \tag{3}$$

Our motivation is that we want to encode the inner structure of patch by cyclic shift. From the above equation, it can be seen that the estimate $\hat{\mathbf{u}}$ is fully depended on the kernel matrix K. Note that K is a large $n \times n$ matrix, and the computational burden in matrix calculation is prohibitively high. However, if weight function κ is chosen as a Gaussian kernel, then the matrix K with elements $K_{i,j}$ is a circulant matrix [30]. That is because $K_{i,j} = \kappa(T^i\mathbf{u}, T^j\mathbf{u}) = \kappa(\mathbf{u}, T^{j-i}\mathbf{u})$, and $K_{i,j}$ depends only on $(j-i) \bmod n$. In addition, Gaussian kernel is easier to adjust than other kernels (i.e., polynomial kernel), since it has only one parameter with an intuitive meaning. Based on the theory of circulant matrices [31], it is clear that the K contains at most n distinct elements and therefore it is often denoted by

$$K = C(\mathbf{k}) = C(k_0, k_1, \ldots, k_{n-1}) \tag{4}$$