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Analyzing the combination of conflicting belief functions

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Abstract

We consider uncertain data which uncertainty is represented by belief functions and that must be combined. The result of the combination of the belief functions can be partially conflictual. Initially Shafer proposed Dempster's rule of combination where the conflict is reallocated proportionally among the other masses. Then Zadeh presented an example where Dempster's rule of combination produces unsatisfactory results. Several solutions were proposed: the TBM solution where masses are not renormalized and conflict is stored in the mass given to the empty set, Yager's solution where the conflict is transferred to the universe and Dubois and Prade's solution where the masses resulting from pairs of conflictual focal elements are transferred to the union of these subsets. Many other suggestions have then been made, creating a 'jungle' of combination rules. We discuss the nature of the combinations (conjunctive versus disjunctive, revision versus updating, static versus dynamic data fusion), argue about the need for a normalization, examine the possible origins of the conflicts, determine if a combination is justified and analyze many of the proposed solutions.

Keywords: Belief functions; Transferable belief model; Combination rules; Conflict; Belief normalization

1. Introduction

The transferable belief model (TBM) is a model for the quantified representation of epistemic uncertainty, i.e., the beliefs (or weighted opinions) held by a 'belief holder', called You hereafter and which can be a robot, an intelligent sensor, etc. Beliefs are represented by belief functions and, among others, by their related basic belief assignments (bba). Familiarity with belief function theory is assumed (see [98] for a recent updated survey).

Our paper concerns the models where belief functions are used to represent beliefs and that do not ask for an explicit underlying probability function. This covers the TBM [104] but also the model presented in Shafer's seminal book [79] which is essentially equal to the TBM except for the normalization phase we will study in this paper. Nevertheless many comments could be applied to the Dempsterian models, i.e., the model studied by Dempster [17,18], those considered in Shafer's papers, the hint model [50], the probabilistic assumption model (PAS) [34]. These Dempsterian models[1] are based on a meaningful[2] underlying probability function, i.e. probabilities which admit an operational definition like those based on betting ratios.

The TBM assumes that there is an object *ob* and an attribute X which depends on *ob*, whose actual value is denoted ω_0 and is supposed to belong to a finite set of possible alternatives denoted $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$. The actual value ω_0 is not known, only beliefs, weighted opinions,

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[1] We do not use the term Dempster–Shafer's theory as it is confusing. Usually the term covers the Dempsterian models, but some authors use it for an upper and lower probability theory or a random set model or a probabilistic model extended to modal propositions.

[2] To be 'meaningful', a probability function must concern a variable on which either bets could be established and settled (for the subjectivists) or frequencies of occurrence could be defined (for the frequentists).

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about its possible value are available. The operational definition of these beliefs is detailed in [100].

Assuming that ω_0 is necessarily an element of Ω is called the closed world assumption. Assuming the possibility that ω_0 might not be any of the alternatives listed in Ω is called the open world assumption. Its meaning is detailed in [87].

The belief about the value of ω_0 is commonly produced by a given source and based on a piece of evidence collected by this source. Suppose the beliefs about ω_0 are produced by two unrelated sources that use two distinct pieces of evidence. These two beliefs are then commonly combined by a conjunctive rule, the operator that performs the data fusion. Shafer proposes Dempster's rule of combination [79]. Then Zadeh presents an example where this rule produces results usually judged unsatisfactory [114–116]. One explanation was that the conflict was mismanaged by Dempster's rule of combination. Since Zadeh's paper, a jungle of alternative rules has bloomed, often leaving the user completely confused.

In fact most rules share the same basis. Suppose two bbas m_1 and m_2 defined on the frame of discernment Ω and let m_{12} be the result of their combination. Let the function $f_{12}: 2^\Omega \to [0,1]$ with:

$$f_{12}(X) = \sum_{A\cap B=X} m_1(A)m_2(B) \quad \forall X \subseteq \Omega. \tag{1}$$

We call it the conjunctive rule. Of course, the issue is to determine m_{12}. In the TBM, m_{12} is equated to f_{12}, in which case $m_{12}(\emptyset)$ may be positive. For Dempster's rule of combination, $f_{12}(\emptyset)$ is proportionally reallocated and:

$$m_{12}(X) = f_{12}(X) + f_{12}(\emptyset)\frac{f_{12}(X)}{1-f_{12}(\emptyset)} = \frac{f_{12}(X)}{1-f_{12}(\emptyset)}.$$

Most other suggested rules are also based on f_{12} and differ on the way they handle $f_{12}(\emptyset)$.

The purpose of this paper consists in detailing the assumptions that underlie the applicability of the combination rules. The correct understanding of these requirements can be used in order to build an expert system that verifies the applicability of the combination rules. Trying to fore a combination of two bbas that do not satisfy the applicability criteria is of course erroneous. We also survey many of the methods proposed to handle conflicts encountered when combining two belief functions, provided the applicability criteria are satisfied. We also study the essential properties they satisfy or not.

In [90], we already discuss the issue but new results justify their reconsideration. Recently [78] present a good survey of the combination rules. We add here some unconsidered rules. More importantly, we examine the assumptions required by the combination.

In this paper, we present the necessary background material in Section 2. In Section 3 we discuss several issues related to the combination and the normalization of bbas. In Section 4, we present the difference between static and dynamic fusion and the consequence of this distinction on the choice of a combination rule. In Section 5, we explain what are the assumptions that underlie the conjunctive combination rules. In Section 6, we explain how we could handle the conflict. In Section 7, we conclude. In Appendix A, we summarize several technical points and list most combination rules discussed in this paper.

We realize that this paper may seem polemical at some places. It raises many issues for which we often present our own view, but we acknowledge that we are of course biased in favor of the TBM. Still we try to stay fair and skeptical. In any case, we did not want to hurt anybody and apologize to those who feel offended.

2. Background material

Some knowledge of the TBM and belief function theory is assumed. We list a few definitions to avoid misunderstanding (Section 2.1) and fix the notation (Section 2.2). We also discuss the concept of source reliability and its related discounting (Section 2.3). Up to date details on the TBM can be found in [98].

2.1. Some definitions

Definition 2.1 (*Frame of discernment*). The frame of discernment is a finite set of mutually exclusive elements, denoted Ω hereafter.

Beware that the frame of discernment is not necessarily exhaustive. Infinite frames of discernment [101] are not used in this paper.

Definition 2.2 (*Basic belief assignment*). A basic belief assignment (bba) is a mapping m^Ω from $2^\Omega \to [0,1]$ that satisfies $\sum_{A\subseteq\Omega} m^\Omega(A) = 1$. The basic belief mass (bbm) $m(A)$, $A \subseteq \Omega$, is the value taken by the bba at A.

Definition 2.3 (*Set of bbas*). The set $\mathscr{B}^\Omega$ is the set of bbas defined on Ω.

Definition 2.4 (*Focal elements*). The focal elements of a bba m^Ω are the subsets A of Ω such that the bbm $m^\Omega(A)$ is positive.

Definition 2.5 (*Categorical belief function*). A categorical belief function on Ω focused on $A^* \subseteq \Omega$, is a belief function which related bba m^Ω satisfies:

$$m^\Omega(A) = \begin{cases} 1 & \text{if } A = A^*, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

When all bbas are categorical, the TBM becomes equivalent to classical propositional logic. Two limiting cases of categorical bbas have received special names.

Definition 2.6 (*Vacuous belief function*). The vacuous belief function on Ω is a categorical belief function focused on Ω. It is denoted VBF.