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A minimal solution for relative pose with unknown focal length

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Abstract

Assume that we have two perspective images with known intrinsic parameters except for an unknown common focal length. It is a minimally constrained problem to find the relative orientation between the two images given six corresponding points. To this problem which to the best of our knowledge was unsolved we present an efficient solver. Through numerical experiments we demonstrate that the algorithm is correct, numerically stable and useful. The solutions are found through eigen-decomposition of a 15×15 matrix. The matrix itself is generated in closed form.

Keywords: Computer vision; Relative pose; Relative orientation; Camera calibration

1. Introduction

The task of computing a 3D reconstruction from a video sequence is central in computer vision. Different paradigms have been proposed for performing this task and the concept of RANSAC has been quite successful [8]. State-of-the-art real-time structure from motion uses the five-point method, e.g. [13], as a RANSAC engine. While this has proved to be efficient and stable, the cameras need to be pre-calibrated. For uncalibrated cameras, the seven-point method can be applied, e.g. [22], but it is not as stable due to projective degeneracies. We offer an attractive compromise having similar performance characteristics as the five-point method and still allowing for unknown focal lengths. By assuming constant and unknown focal lengths, but otherwise known intrinsics, a minimal problem arises for six points. This situation occurs if we assume the principal point to be in the middle of the image, that there is no skew and the aspect ratio is one. A detailed analysis of this case is given, showing the number of possible solutions, efficient ways to compute them and the stability of the solution with respect to measurement noise (Fig. 1).

Minimal case solvers have been built for a large number of camera models. The problem for two calibrated cameras and five points was first solved by Kruppa [11] who claimed 11 solutions (one too many) and the false root was then eliminated by Faugeras and Maybank [4]. A practical solution was given in [15] and improved in [13]. For three views and four points, the problem is not minimal but as it would be under-constrained with three points, the problem is still of interest and was solved in [14]. Minimal solutions also exist for uncalibrated perspective cameras [8], and uncalibrated affine cameras [9].

Given that the epipolar geometry has been computed in terms of the fundamental matrix, it is well known that it is possible to recover the focal length [8,21,10]. However, to the best of our knowledge the relative pose problem for minimal data is still unsolved. Here we present a solver for two cameras and six points. The solver constructs a 15×15 matrix in closed form. Solving the eigen-problem for this matrix gives the 15 (possibly complex) solutions to the relative pose problem.

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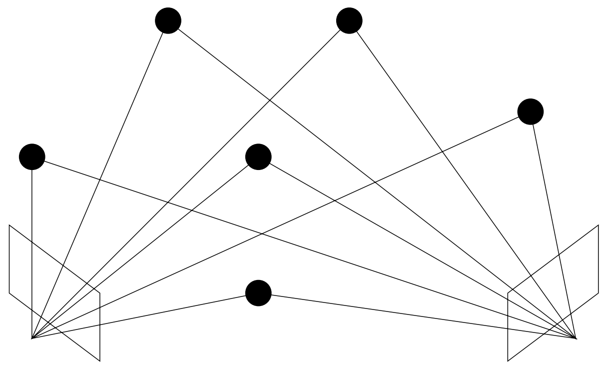

Fig. 1. The problem solved here: Relative orientation for two cameras, with a common but unknown focal length f, that see six unknown points.

We will first list the minimal cases for cameras with a common unknown focal length. Then the equations used will be introduced. The solver was found using Gröbner basis theory but this theory is not necessary to understand the solver. We will also give some numerical results.

2. Background

Suppose we are given m cameras, all calibrated except for a common unknown focal length and n corresponding image points. There are $6m + 3n + 1 - 7$ degrees of freedom (6 for each camera, 3 for each point, 1 for focal length and 7 for the unknown coordinate system) and $2mn$ equations, hence in total, there are $2mn + 6 - 6m - 3n$ excess constraints (Table 1).

The minimal case $(m, n) = (2, 6)$ will be solved here. The other possibility $(m, n) = (3, 4)$ is still an open problem.

2.1. Geometric constraints

The fundamental matrix F encodes the epipolar geometry of two views, and corresponding image points x and x' satisfy the coplanarity constraint

$$x^{\mathrm{T}} F x' = 0. \tag{1}$$

Any rank-2 matrix is a possible fundamental matrix, i.e. we have the well known single cubic constraint, e.g. [8]:

Theorem 1. *If a real non-zero 3×3 matrix F is a fundamental matrix then*

$$\det(F) = 0. \tag{2}$$

Table 1
Number of excess constraints for m views and n points with unknown focal length f

m	n						
	1	2	3	4	5	6	7
1	−1	−2	−3	−4	−5	−6	−7
2	−5	−4	−3	−2	−1	**0**	1
3	−9	−6	−3	**0**	3	6	9
4	−13	−8	−3	2	7	12	17

An essential matrix has the additional property that the two non-zero singular values are equal. This leads to the following cubic constraints on the essential matrix, adapted from Maybank [12]:

Theorem 2. *A real non-zero 3×3 matrix E is an essential matrix if and only if it satisfies the equation*

$$2EE^{\mathrm{T}}E - \mathrm{tr}(EE^{\mathrm{T}})E = 0. \tag{3}$$

This constraint previously appeared in [19,3].

2.2. Gröbner bases

Here are only given some basic notions about algebraic geometry, the interested reader should consult [1,2].

The *ideal* generated by polynomials $f_1, \ldots, f_n \in \mathbb{C}[x_1, \ldots, x_n]$ is the set I of polynomials $g \in \mathbb{C}[x_1, \ldots, x_n]$ of the form:

$$g = \sum_{i=1}^{n} f_i p_i, \quad p_i \in \mathbb{C}[x_1, \ldots, x_n]. \tag{4}$$

We also say that the f_i *generate* the ideal I. A *Gröbner basis* of an ideal is a special set of generators, with the property that the leading term of every ideal element is divisible by the leading term of a generator. The notion of leading term is defined relative to a *monomial order*. The Gröbner basis exposes all leading terms of the ideal and leads to the useful notion of *remainder* with respect to (division by) the ideal. Gaussian elimination is a special case of *Buchberger's algorithm* which is a method for calculating a Gröbner basis from any generating set. Gröbner bases, monomial order and Buchberger's algorithm are explained in [1]. For ideals having a finite set of solutions ("zero-dimensional" ideals) the (vector space) dimension of the quotient ring $A = \mathbb{C}[x_1, \ldots, x_n]/I$ is also finite and the dimension equals the number of solutions, counted with multiplicity. Any polynomial f acts on the quotient ring A by multiplication ($f : g + I \mapsto fg + I$) and this is clearly a linear mapping from A to itself. A natural way to choose a (vector space) basis for A is to take all monomials that are not leading terms of any element of I. The action of a polynomial f is then described by a square matrix m_f called the *action matrix*. If R/I is finite and I has n solutions, the action matrix for multiplication by the polynomial f is computed by concatenating the $n \times 1$ vectors formed by taking the n elements in the monomial basis of R/I, reducing modulo gb(I) and representing them as $n \times 1$ vectors in the basis of R/I. The solutions to a zero-dimensional ideal can be read off directly from the left eigen-values and left eigen-vectors of appropriate action matrices [2].

The solver presented here was built by first computing a Gröbner Trace over $\mathbb{Z}_p$, please see [23,25]. The solver we build in this paper has some similarity with the matrix based Gröbner base algorithm proposed in [6]. A description of the machinery we used to build this solver is given in [20]. For the work in the finite field we used [7].