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A fast scanning based message receiving method on four directed acyclic subgraphs ☆

Jeongmok Ha *, Hong Jeong

Department of Electrical Engineering, Pohang University of Science and Technology, Pohang, South Korea

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ABSTRACT

We propose a message-receiving algorithm on a Directed Acyclic Subgraph (DAS) structure to approximate the solution of general labeling problems extremely quickly. The algorithm divides a graph into four subgraphs to get a joint distribution of all nodes, then passes messages in two fixed directions as inference on DASs. Message receiving is a modified version of message passing. When receiving messages on DAS structure, labeling results can be obtained after just four scans. The proposed algorithm was evaluated by using it to perform three labeling decision applications (binary segmentation, image denoising, and stereo matching). Compared to other highly-accurate iterative algorithms (α-expansion, α-β swap, tree-reweighted message passing, sum–product belief propagation, max–product belief propagation, and FastPD), the proposed algorithm shows competitive accuracy but requires much less computational time. The proposed algorithm is appropriate for applications in which iterative schemes are undesirable, but which must get reliable labeling results within a limited time.

1. Introduction

Most low-level to mid-level computer vision and image processing problems can be represented as labeling problems [1,2]. By importing meaning into labels of each pixel, valuable information can be obtained by solving labeling problems. For computer vision and image processing, the labeling problem is solved by finding the optimum label set from given images. Numerous applications can be solved as labeling problems [3–5], e.g., image segmentation [6–9], image restoration [10–12], background subtraction [13–15], and stereo matching [16–18].

To get an exact labeling solution, all possible label sets must be tried. However, this is an NP-hard problem, which means that it is computationally intractable in most cases; therefore, authors frequently seek an approximate solution instead of the exact solution. One possible way to find the approximate labeling solution is to denote labels as random variables on the *Markov random fields* (MRFs) model [19]. The MRF model is an undirected graphical model that consists of a set of random variables that satisfy Markov properties on an undirected graph. The MRF model helps to obtain an approximate solution in a reasonable time, so this model can be used as a practical answer.

Inference techniques for the MRF model have been compared thoroughly [20,21]. Approximate inference on the MRF model is usually based on an iterative scheme that generates lower energy at each iteration [22]. The approximate solution is estimated by repeating local optimization on pre-defined neighbor structure. *Graph cuts* (GC) [23–25], *belief propagation* (BP) [26,27], and *tree-reweighted message passing* (TRW) [28,29] are popular iterative algorithms [30]. GC is a graph-partitioning algorithm for general labeling problems that iterates in the label space until maximum flow converges. Both BP and TRW use message passing to update a local distribution among the neighbors by sharing its messages iteratively.

These iterative algorithms give accurate approximate labeling solutions, but require high computational cost to get reliable labeling results. Standard iterative algorithms cannot be implemented in real-time processing systems that do not use parallel computational processors e.g., *field programmable gate arrays* (FPGA) [31–38] or *graphic processing units* (GPU) [39–43]. For this reason, various efficient methods [44–47] or iteration-based algorithms related to parallel architecture have proposed to reduce the time required to obtain reliable labeling results. Even though iterative algorithms based on parallel structure can produce output quickly, implementing them on the parallel processers remains a difficult and costly task.

In addition to computational complexity, algorithms based on iterative schemes have basic weaknesses that impede their

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* Corresponding author.
E-mail address: jmokha@postech.ac.kr (J. Ha).

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practical use [48]. First, iterative schemes are not guaranteed to converge on a solution in time. This is a very important limitation in low-level vision systems treated as preprocessing [49]. Failure to guarantee a maximum operation time restricts the application of iterative methods in various applications. Second, iterative algorithms need an initial condition which sometimes has a huge influence on convergence time, so initial labels must be set appropriately, but choosing a suitable initial value of the label for each application is a difficult task.

To circumvent the disadvantages of iterative methods, we consider three topics: efficient graphical model (*directed acyclic subgraphs* (DAS)), efficient inference method (*message receiving*), and efficient algorithm (*scanning*). We propose a new *directed acyclic graph* (DAG) [50–52] structure that consists of *directed* edges so that the scanning scheme can be adjusted; this structure differs from that used in the standard MRF model; i.e., that uses *undirected* edges. For the optimization process, joint distribution of all nodes must be defined before the inference process. To obtain the joint distribution of all nodes in the image, we propose a DAS graphical structure, which is a set of subgraph structures defined by dividing the fully-connected DAG structure into four subgraph structures. Four subgraphs are used to construct the full graph structure because each subgraph covers one quarter of the image.

For an inference on the four DAS structures, we propose a *message receiving* method, which is a modified version of *message passing* [26]. Whereas standard message passing delivers information to neighbor nodes iteratively, *message receiving* is focused on collecting information from neighbor nodes. With the help of message receiving, inference on the DAS structure generates labeling results much more quickly than do iterative methods; also, it does not need any initial or stopping conditions.

We also propose a fast *scanning*-based method on the proposed DAS structure. To estimate an approximate global energy-minimization solution using scanning, the method scans the local minimum just once from one corner to the opposite corner in a horizontal or vertical direction while considering neighbor relations. The scanning method needs only one computation on each node to obtain the optimum lowest energy area, so this method usually gives the optimal solution quickly. In this proposed work, our algorithm requires four scans for each DAS to get reliable labeling results.

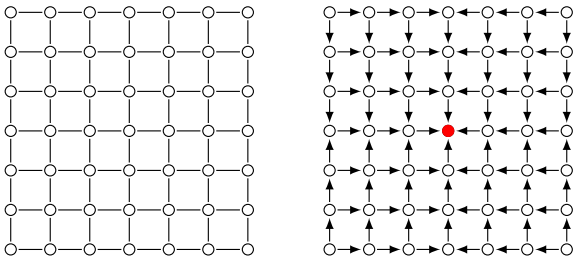
This paper is organized as follows: In Section 2, the general labeling problem on MRF model is introduced. In Section 3, a new DAS graph structure is proposed. In Section 4, an inference on DAS structure is proposed. In Section 5, a fast four-scanning algorithm is proposed. In Section 6, accuracy and computation time of proposed algorithm are compared to those of high-performing iterative methods. In Section 7, parameter sensitivity is analyzed. In Section 8, the proposed algorithm is summarized.

2. Markov random fields

In the proposed work, the considered MRF model is a four-connected *undirected cyclic graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is a set of nodes and $\mathcal{E}$ is a set of undirected edges between them (Fig. 1a). A node $\mathbf{p} \in \mathcal{V}$ is a random variable in $\mathcal{V}$, and has label l drawn from a set of labels $\mathcal{L}$. We define $\mathbf{l}$ as a label array and $\mathbf{o}$ as observed data. The labeling problem on $\mathcal{G}$ is solved by estimating labels that maximize the joint distribution of labels given observed data: $\mathbf{l}^* = \arg\max_{\mathbf{l}} P(\mathbf{l}|\mathbf{o}, \mathcal{G})$, where $\mathbf{l}^*$ is an optimal label array.

The distribution of label array given observed data is determined by product of likelihood and prior distributions:

$$P(\mathbf{l}|\mathbf{o}, \mathcal{G}) = P(\mathbf{o}|\mathbf{l}, \mathcal{G}) P(\mathbf{l}|\mathcal{G}); \tag{1}$$



(a) Undirected graph $\mathcal{G}$ (b) Directed graph $\vec{\mathcal{G}}$

Fig. 1. Graph structure comparison; (a) graph with undirected edges (lines) and (b) graph with directed edges (arrows).

this is called *maximum a posteriori* (MAP) estimation. By using Gibbs models [53], this stochastic formulation is changed to an energy minimization formulation:

$$E(\mathbf{l}) = \sum_{\mathbf{p} \in \mathcal{V}} \phi_{\mathbf{p}}(l_{\mathbf{p}}) + \sum_{(\mathbf{p},\mathbf{q}) \in \mathcal{E}} \psi_{(\mathbf{p},\mathbf{q})}(l_{\mathbf{p}}, l_{\mathbf{q}}), \tag{2}$$

where $l_{\mathbf{p}}$ is a label value on node $\mathbf{p}$, $\phi_{\mathbf{p}}(\cdot)$ is a unary data term that represents likelihood distribution on node $\mathbf{p}$, and pairwise term $\psi_{(\mathbf{p},\mathbf{q})}(\cdot,\cdot)$ is a smoothness term that represents prior distribution on edge $(\mathbf{p}, \mathbf{q})$. Now the goal is to find optimum label array which makes energy minimum: $\mathbf{l}^* = \arg\min_{\mathbf{l}} E(\mathbf{l})$.

Because optimizing the global solution is NP-hard problem [54], solving it can entail enormous computational cost. Usually, the labeling problem on the MRF model is solved using an iterative scheme based on approximated energy minimization formulation:

$$E(\mathbf{l}) = \sum_{\mathbf{p} \in \mathcal{V}} E(l_{\mathbf{p}}) = \sum_{\mathbf{p} \in \mathcal{V}} \left[\phi_{\mathbf{p}}(l_{\mathbf{p}}) + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} \psi_{(\mathbf{p},\mathbf{q})}(l_{\mathbf{p}}, l_{\mathbf{q}}) \right], \tag{3}$$

where $\mathcal{N}_{\mathbf{p}}$ is a set of neighbor nodes of $\mathbf{p}$. These methods approximate the global minimum by optimizing local minima iteratively.

3. Directed acyclic subgraphs

In this paper, the idea of finding the optimum label is not based on the MRF model. We propose a fast labeling algorithm by changing the undirected MRF network to directed Bayesian subnetworks. Optimizing the undirected network requires an iterative procedure that optimizes local minima by sharing information from their neighbors or by changing label candidates. So to increase the speed of the algorithm we restrict the direction of sharing information on the belief network. Compared to iterative methods, the proposed method gives a slightly less accurate solution but is much faster. Another idea is that we divide the four-connected full grid graph into four subgraphs; this division enables efficient message passing.

In this paper, we solve the labeling problem on the following graph structure. We denote $\vec{\mathcal{G}} = (\vec{\mathcal{V}}, \vec{\mathcal{E}})$ as a *directed acyclic graph* (DAG) that consists of a set of nodes $\vec{\mathcal{V}}$ (same as $\mathcal{V}$) and a set of directed edges $\vec{\mathcal{E}}$ (Fig. 1b), in a manner homologous to the structure of undirected graph $\mathcal{G}$. In both the undirected graph and the DAG (Fig. 1), every node is four-connected to its neighbor nodes; however, in $\vec{\mathcal{G}}$, each edge between nodes has only one direction. On this directed graph, the optimization process based on iterative message passing can be avoided.

The goal of this paper becomes one of estimating optimum labels that maximize the joint distribution of labels given observed data on $\vec{\mathcal{G}}$: $\mathbf{l}^* = \arg\max_{\mathbf{l}} P(\mathbf{l}|\mathbf{o}, \vec{\mathcal{G}})$. This problem can also be solved by MAP estimation which is similar to inference on the MRF model: