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Fast total variation deconvolution for blurred image contaminated by Poisson noise ☆

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ABSTRACT

In this paper, we present a fast non-blind deconvolution method for restoring blurred images contaminated by Poisson noise. The problem is formulated by finding the minimizer of the negative logarithmic Poisson log-likelihood combined with the total variation (TV). To attack the challenging task, we adopt the well-known variable splitting and penalty technique to convert the problem into two easier sub-problems: one is a modified TV regularized deconvolution and the other is a simple convex optimization problem. Experimental results show that the proposed method runs very fast and the quality of the restored image is comparable with that of some state of the art methods.

1. Introduction

Blurring is one of the most common phenomena that degrade the quality of the obtained images. In mathematics, it is formulated by convolving the latent image with a point spread function (PSF), besides, due to the measurement error, some noise must be added, i.e.,

$$\mathbf{g} = \mathbf{ho} + \mathbf{n} \tag{1}$$

where $\mathbf{g}$, $\mathbf{o}$ and $\mathbf{n}$ represent the vector forms of the blurred image, latent image and noise, respectively, $\mathbf{h}$ denotes the convolution matrix of the PSF. The noise $\mathbf{n}$ is a random variable which is usually modeled by the Gaussian or the Poisson distribution.

In frequency domain, Eq. (1) is converted into the following equation,

$$G = H \bullet O + N \tag{2}$$

where G, H, O and N are the discrete Fourier transforms of $\mathbf{g}$, $\mathbf{h}$, $\mathbf{o}$ and $\mathbf{n}$, respectively, the operator $\bullet$ stands for the component-wise multiplication.

Image deconvolution is the inverse process of image blurring. The algorithms can be divided into two categories in terms of whether the PSF is known, i.e., non-blind and blind image deconvolution. In some special applications such as remote sensing, the PSF can be measured in advance and the only unknown variable that needs to be solved is the latent image [1,2], which is non-blind deconvolution. However, in most cases, we have to estimate both the PSF and the latent image from the blurred image and this is the implication of blind deconvolution.

Unfortunately, even non-blind image deconvolution is inherently ill-posed, which means it is very difficult to obtain a restored image free of noise and ringing artifacts. This is mainly because the PSF is low pass in frequency domain, while the main components of the noise and the Gibbs effect concentrate in the high frequency region, they tends to be amplified in the direct inverse process [3–5]. In blind image deconvolution, the PSF is unknown and the situation will be even worse [6]. To obtain a stable solution, various regularized methods have been proposed, e.g., the Tikhonov regularization [7–9], the total variation (TV) regularization [10–14] and the regularization using sparse priors [15–17].

Till now, most of the researches consider the Gaussian noise model which results in a quadratic fidelity term in the optimization

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problem [18]. However, in some fields such as the astronomical observation and the medical imaging, the noise follows Poisson distribution [19], and it is unreasonable to be modeled with the Gaussian distribution. The most commonly used approach to restore the blurred images contaminated by Poisson noise is the classic Richardson–Lucy (RL) algorithm [4,5] which is essential an special case of the Expectation–Maximization (EM) method [20]. Just as mentioned above, due to the ill-posedness of the problem, the result of the RL algorithm is usually of low quality, thus regularization is necessary.

However, to solve the resulted regularized deconvolution problem under the condition of Poisson noise is not an easy task. In [21,22], the authors propose to use the EM algorithm and until now it is still the most widely used scheme. E.g., in [23], the authors propose to use the Tikhonov regularization. In [14,24,25], the authors use the TV regularization to restore the blurred microscope image. In [26], the authors propose an algorithm with an adaptive TV regularization. While in [27], a TV regularized step is integrated into each EM step to suppress the negative artifacts. In [28], the sparse natural image gradient prior is used for regularization. In [29], the authors design a Gaussian Markov random field for regularization and adopt a piecewise function to adjust the penalty strength. In [30], the authors adopt a Gaussian distribution to model the differences between the gradients of the blurred and latent images and combine it with an edge mask to suppress the negative artifacts. In [31], the Huber–Markov random field regularization is used. All the listed methods depend on the EM algorithm. However, experimental results show that they cannot always converge to good results and are not of high executive efficiency. One popular way for speeding up is to use parallel computing enabled by many-core processers [32–34]. However, we focus on designing new efficient algorithm in this paper.

In recent years, some very efficient optimization algorithms have been introduced for image restoration under the assumption of Gaussian noise. E.g., in [35,36], the authors respectively design the Bregman iterative algorithm and the Split Bregman algorithm to solve the l_1-norm regularized problems in image denoising and compressed sensing. To attack the more difficult problem due to Poisson noise, some researchers have tried to draw on ideas from these methods. In [37], the authors use the split Bregman algorithm, while in [38,39], the variable splitting and augmented Lagrangian algorithm is adopted. The two methods are of high efficiency and the restored images are of high quality, which makes them become the foundation and guidance of many recent researches [40–45].

In this paper, we focus on the non-blind TV regularized deconvolution and design a very efficient method to restore the blurred image contaminated by Poisson noise. In [46], the authors propose an optimization algorithm which adopts the well-known variable-splitting and penalty technique, it runs very fast and can reach restored image with very high quality. In [16], the same algorithm is applied to the l_p-norm ($0.5 \leqslant p \leqslant 0.8$) regularized deconvolution and proven to be very successful. We learn from the approach and use the variable splitting technique to divide the problem into two sub-problems: one is a modified version of the TV regularized deconvolution presented in [46], and the other is a simple convex optimization problem. Experimental results show that the proposed method runs very fast, with only a few iterations it can reach results which is comparable with that of some state of the art methods.

The arrangement of the paper is as follow. In Section 2, we formulate the problem to be solved and make a necessary introduction to some important related methods. In Section 3, we make a detailed description of the proposed algorithm. In Section 4, we take experiments to compare our approach with some state of the art methods. Finally in Section 5, a conclusion is made.

2. Problem formulation and related work

In Bayesian probabilistic framework, non-blind image deconvolution can be modeled by maximum a posteriori (MAP) estimation of the latent image given the blurred, i.e.,

$$\begin{aligned}\mathbf{o} &= \arg\max_{\mathbf{o}} P(\mathbf{o}|\mathbf{g}) \\ &= \arg\max_{\mathbf{o}} P(\mathbf{g}|\mathbf{o})P(\mathbf{o})\end{aligned} \tag{3}$$

where the terms $P(\mathbf{g}|\mathbf{o})$ and $P(\mathbf{o})$ denote the prior distributions of noise and the latent image, respectively. With negative logarithmic operation, Eq. (3) is converted into the following equation:

$$\mathbf{o} = \arg\min_{\mathbf{o}} \{-\ln[P(\mathbf{g}|\mathbf{o})] - \ln[P(\mathbf{o})]\} \tag{4}$$

In view of the topic of the paper, we adopt the Poisson noise model and suppose that the elements of the random variable $\mathbf{n}$ are independent and identically distributed, then

$$P(\mathbf{g}|\mathbf{o}) = \prod_{i=1}^{N} \frac{\exp[-(\mathbf{ho})_i](\mathbf{ho})_i^{\mathbf{g}_i}}{\mathbf{g}_i} \tag{5}$$

$$-\ln[P(\mathbf{g}|\mathbf{o})] = L(\mathbf{o}) = \sum_{i=1}^{N}\{(\mathbf{ho})_i - \mathbf{g}_i \ln[(\mathbf{ho})_i] + \ln(\mathbf{g}_i)\} \tag{6}$$

where i is the element index, N denotes the total number of the elements in each variable.

To model the term $-\ln[P(\mathbf{o})]$, the TV regularization is adopted, i.e.,

$$-\ln[P(\mathbf{o})] = TV(\mathbf{o}) = \sum_{i}^{N}\sqrt{(\mathbf{d}_1\mathbf{o})_i + (\mathbf{d}_2\mathbf{o})_i} \tag{7}$$

where $\mathbf{d}_1$ and $\mathbf{d}_2$ represent convolution matrices of the horizontal and vertical derivative operators, respectively.

Bringing Eqs. (6) and (7) into Eq. (4), we obtain that

$$\mathbf{o} = \arg\min_{\mathbf{o}} \frac{\lambda}{2}\sum_{i=1}^{N}\{(\mathbf{ho})_i - \mathbf{g}_i \ln[(\mathbf{ho})_i]\} + \sum_{i}^{N}\sqrt{(\mathbf{d}_1\mathbf{o})_i + (\mathbf{d}_2\mathbf{o})_i} \tag{8}$$

Furthermore, just like in [37,38], we introduce an indicator function $\iota_{R_N^+}(\mathbf{o})$ to impose the non-negativity constraint on the estimate, i.e.,

$$\iota_{R_N^+}(\mathbf{o}) = \begin{cases} 0 & \textit{if } \mathbf{o} \in R_N^+ \\ +\infty & \textit{if } \mathbf{o} \notin R_N^+ \end{cases} \tag{9}$$

The final optimization problem to be solved is expressed by the following equation, which is proper, lower semi-continuous, and convex [38].

$$\mathbf{o} = \arg\min_{\mathbf{o}} \frac{\lambda}{2}\sum_{i=1}^{N}\{(\mathbf{ho})_i - \mathbf{g}_i \ln[(\mathbf{ho})_i]\} + \sum_{i}^{N}\sqrt{(\mathbf{d}_1\mathbf{o})_i + (\mathbf{d}_2\mathbf{o})_i} + \iota_{R_N^+}(\mathbf{o}) \tag{10}$$

Since in Section 4, we will compare our approach with the methods in [37,38,42], and also because the three methods are of many similarities, here we make some brief descriptions to help the readers understand the difference between them.

In [37,38], the authors use the split Bregman method and the augmented Lagrangian method to solve the problem in Eq. (10) and named their optimization schemes *PIDSplit+* and *PIDAL-TV*, respectively. However, because of the close relationship between the split Bregman method and the augmented Lagrangian method, the optimization procedures are similar, the difference concentrates upon the strategies of how to convert the complicated problem into easier sub-problems. In Figs. 1 and 2, the iterative steps of the two algorithms are given, we can see that they all