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Tri-directional gradient operators for hexagonal image processing ☆

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ABSTRACT

Image processing has traditionally involved the use of square operators on regular rectangular image lattices. For many years the concept of using hexagonal pixels for image capture has been investigated, and several advantages of such an approach have been highlighted. We present a design procedure for hexagonal gradient operators, developed within the finite element framework, for use on hexagonal pixel based images. In order to evaluate the approach, we generate pseudo hexagonal images via resizing and resampling of rectangular images. This approach also allows us to present results visually without the use of hexagonal lattice capture or display hardware. We provide comparative results with existing gradient operators, both rectangular and hexagonal.

1. Introduction

In machine vision, feature detection is often used to extract salient information from images. Image content often represents curved structures, which increase the complexity of the information to be processed compared with structures that can be described by a discrete set of directions. Most well-known operators on a conventional rectangular lattice exhibit limitations when detecting curved edges, most commonly due to the processing being aligned principally in the horizontal and vertical directions. Potentially image information is excluded or lost due to failure to represent and process curves accurately [24]. One method to improve the treatment of curved objects is the use of compass operators that rotate feature detection masks to successfully detect diagonal edges [19]. An alternative approach is to increase the image resolution if possible [15]. This can help to reduce the loss of information, but increase in image resolution generally leads to an increase in computational cost. To overcome this problem, an alternative sampling lattice, i.e., hexagonal, can be introduced.

Machine vision systems are often modelled on characteristics of the human vision system, in which photoreceptors in the human fovea are densely packed in a hexagonal structure as illustrated in Fig. 1. The characteristics of the human vision system have been used to construct a noise spectrum [3] and Gabor filters [31] for use on a hexagonal grid structure. Applications have been developed including biologically-inspired fovea modelling with neural networks that correspond to the hexagonal biological structure of photoreceptors [9], and silicon retinas for robot vision [17,28]. Indeed, hexagonal lattices have been explored for approximately forty years [8,33,29]; research on processing hexagonal images includes areas such as image reconstruction [33,13], hexagonal filter banks [11], blue-noise halftoning [9], and robot exploration [22].

There is a number of fundamental advantages of using the hexagonal grid structure for digital image representation. One of the major advantages is the consistency available in terms of neighbouring pixel distances when tiling an image plane. In a rectangular grid, the distance d between neighbouring pixel centres depends on whether the neighbours are vertically/horizontally adjacent, (with $d = 1$), or diagonally adjacent (with $d = \sqrt{2}$) as illustrated in Fig. 2(a).

In a hexagonal lattice, the distances between all neighbouring pixels are equal, i.e., $d = 1$ in all cases, as shown in Fig. 2(b). This equidistance property facilitates the implementation of circular symmetric kernels and is associated with increased directional accuracy when detecting edges, both straight and curved [10]. The accuracy of circular and near-circular operators for edge detection has been demonstrated in [27,6]. Sampling on a hexagonal grid has also been shown to incur less aliasing [19] and afford greater efficiency in terms of sampling density than on a square lattice. Vitulli [23] shows that to achieve the same average vertical sampling density (i.e., the same number of pixel rows in the image) 13% fewer pixels are required with hexagonal sampling than with square sampling. Hence, less storage in memory will be needed for the image data, and potentially less computational time to process the image.

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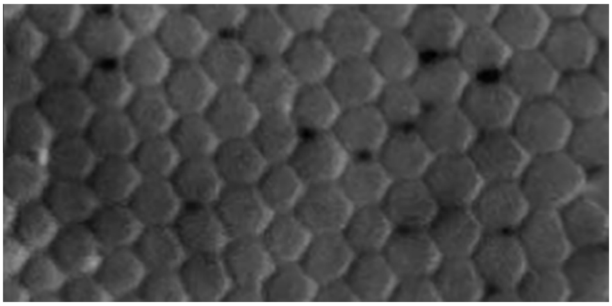

Fig. 1. Cross section of human fovea showing the hexagonal structure of the photoreceptor cones densely packed [5].

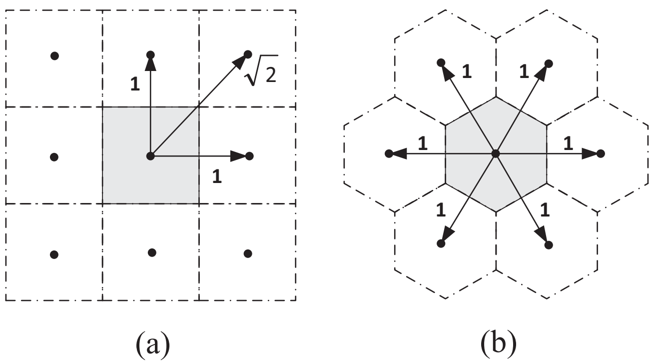

Fig. 2. Adjacency properties of (a) a square lattice and (b) a hexagonal lattice.

Many edge detection algorithms that exist for conventional images are based on components strongly aligned with the horizontal and vertical axes, and hence they are not readily adaptable to a hexagonal lattice. To date, only a small number of hexagonal gradient operators have been designed for use on hexagonal images, such as the work of Davies [6], Middleton and Sivaswamy [15], and Staunton [24]. More recently, Shima et al. [30] designed new consistent gradient operators for direct use on hexagonal images; the approach is based on that in [25] where the operators are derived by minimizing the difference between the ideal gradient response and that obtained by the gradient operator. In addition, recent work has highlighted the advantages of omni-directional feature extraction [26], and in particular, Paplinski [1] has introduced tri-directional feature extraction on traditional rectangular pixel-based images. The use of a hexagonal image structure facilitates tri-directional feature extraction by introducing three natural axes along which directional derivative operators may be easily computed. One of the main advantages of using the three natural axes of symmetry is that directional derivatives along axial directions can be computed efficiently by rotation. We need compute operators for only one specific axial direction (say, x-axis) and then transform the operator through 60° rotations to generate operators along the other two axial directions.

We present an efficient design procedure for the development of hexagonal tri-directional derivative operators that can be applied directly to hexagonal images. We show that only one operator (x-directional derivative) needs to be computed and the other two operators can then be obtained via appropriate rotation. We demonstrate that, unlike the approach of previously developed operators for use on hexagonal images, our design procedure facilitates the development of gradient operators of any size. For example, the operators developed by Davies [6], which use masks designed on the Cartesian axes, are not readily scalable to larger neighbourhood operators; the tri-directional hexagonal operators developed by Shima [30] use Fourier transforms, are quite computationally expensive and do not offer flexibility to readily scale the operator neighbourhood. In Section 3 we show our operator design procedure in detail, demonstrating that only a small number of simple function evaluations are required.

The paper is organised as follows: Section 2 describes the resampling technique that enables an image to be represented using hexagonal pixels. In Section 3 we present the tri-directional operator design, highlighting how this can be readily scalable to operators of any size. In Section 4 we present performance evaluation that illustrates that the proposed gradient operators are superior to the current state-of-the-art techniques when comparing performance over a range of edge orientations and displacements. Our approach, combined with the spatial sampling efficiency of a hexagonal structure, also provides improvements in computational performance in comparison with the use of traditional rectangular pixel-based images. A conclusion is provided in Section 5.

2. Resampling techniques

One of the main restrictions on adoption of the hexagonal lattice for image representation and processing is the absence of availability of hardware: both sensors that enable the capture of hexagonal images and devices that enable their display. In order for research to advance in this area a resampling technique must be incorporated to enable the processing and display of hexagonal images using existing square lattice hardware.

In Gardiner et al. [7], a comparative evaluation was completed to determine the most appropriate resampling technique to generate hexagonal pixel-based images, evaluating those discussed in [34,16,21,4]. Based on the evaluation results obtained in [7], we have chosen to use the resampling technique in [4] throughout this work. Wüthrich and Stucki [4] proposed a method of creating a pseudo hexagonal pixel, known as a hyperpel, from a cluster of square pixels. In [20], this approach is adapted to use sub-pixels to enable a hexagonal pixel to be formed from a cluster of square sub-pixels. Sub-pixel creation limits the loss of image resolution. As illustrated in Fig. 3, each pixel of the original image is represented by a 7×7 pixel block of equal intensity in the new image [20]. This creates a resized image of the same resolution as the original image with the ability to display each pixel as a group of sub-pixels.

The motivation for image resizing is to enable the display of sub-pixels, which is not otherwise possible. With this structure now in place, a cluster of sub-pixels in the new image, closely representing the shape of a hexagon, can be selected; this cluster represents a single hexagonal pixel in the resized image, and its intensity value is the average of the intensity values of its sub-pixels. Selection of the number of sub-pixels to be clustered for each hexagonal pixel is based on two issues: the arrangement must allow a tessellation of the image plane; and the cluster must

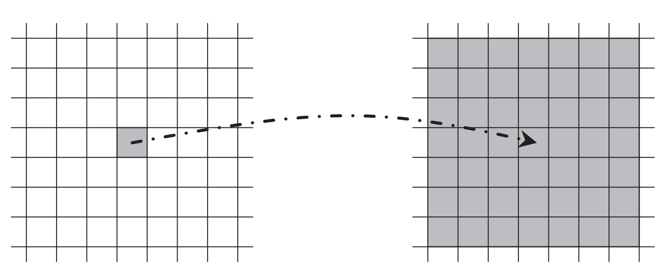

Fig. 3. Resizing of a pixel to a 7×7 pixel block.