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Image representation by harmonic transforms with parameters in $SL(2,R)$ ☆

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ABSTRACT

In this paper, a kind of invariant harmonic transforms with parameters in $SL(2,R)$ are proposed, which include the polar linear canonical transform (PLCT) and the two-dimensional linear canonical transform series (2-D LCTS). The capabilities of the PLCT and the 2-D LCTS on image representation are analyzed. The experimental results show that the 2-D LCTS has much stronger capability on the image representation with respect to characters, and has better invariance of scale and noise than the other transforms. Moreover, due to the varieties of parameters, the performance of the image representation is going bad when parameters used in the reconstructing process are inconsistent with those in the decomposing process. In other words, the proposed transforms can be used for the protection of the image safety because they have free parameters than the traditional methods.

1. Introduction

With the rapid development of the field of computer and communication technology, digital images have become a significant part of our lives. A large number of techniques for image processing and reconstruction have been proposed in recent years. Among them, the invariant moments and transforms have attracted more and more researchers, and successfully been applied in visual pattern recognition, face recognition, character recognition, digital watermarking and edge detection. Readers are encouraged to refer to [1–11] for more details and information.

Let $f(x,y)$ be a given image function, then the moments or transform coefficients are defined as

$$M_{nm} = \int\int_{x^2+y^2\leqslant 1} f(x,y)K^*_{nm}(x,y)dxdy,$$

where K_{nm} is a kernel with respect to integers n and m, and $*$ denotes the complex conjugate. Based on the orthogonality property of the kernel K_{nm}, the moments and transforms are divided into the orthogonal type and the non orthogonal type. The non-orthogonal type includes Fourier-Radial Mellin Descriptors (FRMD) [12], Rotational Moments (RMs) [13] and Complex Moments (CMs) [14,15]; while the orthogonal type includes Zernike Moments (ZMs) [16,17], Pseudo-Zernike Moments (PZMs) [13], Orthogonal Fourier–Mellin Moments (OFMMs) [3,18], Tchebichef Moments (TM) [19] and Radial-Harmonic Fourier Moments (RHFMs) [20]. Because of their good image-describing and image-reconstructing capability, these methods are widely applied in image processing.

In order to overcome the high computational expense and numerical instability issues at the greater moments of traditional moments and transforms, Yap, Jiang and Kot in [22] introduced a new kind of transforms, named the Polar Harmonic Transforms (PHTs), whose kernels are significantly simpler to compute compared with ZMs and PZMs. These transforms have advantages in time complexity and numerical stability, and have been applied in the image watermarking, seeing details in [23,24]. It should be noticed that the PHTs can be seen as the forms based on the theory of the Fourier Transform (FT) which is a special case of the Linear Canonical Transform.

The Linear Canonical Transform (LCT) was introduced in [25] by Moshinsky and Quesne during 1970s, which is a linear integral transform with parameters in a real special linear group $SL(2,R)$, and has been proved to be a powerful tool in several areas, including optics, signal processing, optimal filtering and time–frequency analysis. We encourage the interested reader to consult [26–28,30,29,31,32]. As will be seen that the FT, the fractional Fourier Transform (FrFT), and the Fresnel transform (FRT) are the

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particular cases of the LCT. Until recently, to the best of our knowledge, there are few research papers published about the LCT on the image representation. Therefore, it is worthwhile to explore the image representation associated with the theory of LCT.

In this paper, a kind of harmonic transforms with parameters in a real special linear group $SL(2,R)$ are proposed to improve the capabilities of the existing results on the image representation and the image safety. The capabilities of the new transforms on the image representation are analyzed. Subsequently, the simulation are also presented to verify image representation of the new transforms, and the comparison of the proposed method with the existing results are also given.

2. Preliminary

2.1. Orthogonal moments and transforms

In this subsection, we will present the basic concepts of the orthogonal moments and transforms. For any integers n and m, the moments (or transform coefficients) M_{nm} of a finite function $f(x,y)$ is represented as [14,17,19,21]:

$$M_{nm} = \int_0^{2\pi}\int_0^1 f(r,\theta)K^*_{n'm'}(r,\theta)rdrd\theta,$$

with $K_{nm}(r,\theta) = R_n(r)A_m(\theta)$, where $R_n(r)$ and $A_m(\theta)$ denote the radial component and the angle component, respectively.

Compared with the non-orthogonal moments and transforms, the performance of image representation can be improved by the orthogonal moments and transforms. Thus, we pay more attention on the orthogonal moments and transforms which satisfy the orthogonality condition, i.e.,

$$\int_0^{2\pi}\int_0^1 K_{nm}(r,\theta)K^*_{nm}(r,\theta)rdrd\theta = \int_0^1 R_n(r)R^*_{n'}(r)rdr \times \int_0^{2\pi} A_m(\theta)A^*_{m'}(\theta)d\theta = c_1c_2\delta_{nn'}\delta_{mm'},$$

where $\delta_{nn'} = 1$ if $n = n'$, otherwise $\delta_{nn'} = 0$ ($\delta_{mm'} = 1$ if $m = m'$, otherwise $\delta_{mm'} = 0$), and $c_1,\ c_2$ are constants.

Normally, the angle component is defined as $A_m(\theta) = \exp(im\theta)$, where $i^2 = -1$. Then different definitions of $R_n(r)$ are used to obtain different moments and transforms. For example, the Polar Complex Exponential Transform (PCET) in the PHTs is defined as [22]:

$$M_{nm} = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 f(r,\theta)K^*_{nm}(r,\theta)rdrd\theta, \tag{1}$$

with $K_{nm}(r,\theta) = \exp(i2\pi nr^2)\exp(im\theta)$; and the Polar Cosine Transform (PCT) is given by

$$M^C_{nm} = \Omega_n\int_0^{2\pi}\int_0^1 f(r,\theta)K^{C*}_{nm}(r,\theta)rdrd\theta, \tag{2}$$

with $K^C_{nm} = \cos(\pi nr^2)\exp(im\theta)$, where $\Omega_n = \frac{1}{\pi}$ if $n = 0$, otherwise $\Omega_n = \frac{2}{\pi}$. In [22], Yap et al. showed that the PCT can be viewed as the best transform among the PHTs on the performance of the image representation. In addition, the kernel of the ZMs and the kernel of the PZMs can be respectively defined as

$$K^z_{nm} = \sum_{s=0}^{(n-|m|)/2}\frac{(-1)^s(n-s)!r^{n-2s}}{s!((n-|m|)/2-s)!(n-|m|-s)!}e^{im\theta}, \quad n = 0,1,\ldots,$$

and

$$K^{pz}_{nm} = \sum_{s=0}^{n-|m|}\frac{(-1)^s(2n+1-s)!r^{n-s}}{s!(n+|m|+1-s)!(n-|m|-s)!}e^{im\theta}, \quad n = 0,1,\ldots,$$

where $\frac{n-|m|}{2}$ is an integer and $|m| \leqslant n$.

2.2. LCT

For the sake of convenience, we denote a real special linear group as

$SL(2,R) = \{A \in M(2,R)|\det(A) = 1\}$,

where $M(2,R)$ is the set of 2×2 real matrices. In the following, we always write the matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

as $A = [a,b;c,d]$ for convenience. For any $A \in SL(2,R)$, it is easy to see that its inverse $A^{-1} = [d,-b;-c,a]$ and the determinant $\det(A) = ad - bc = 1$.

The one-dimensional LCT (short for 1-D LCT) with parameter $A = [a,b;c,d] \in SL(2,R)$ of a function $f(x)$ is defined as [33]:

$$L^A_f(u) = \mathcal{L}_A[f](u) = \begin{cases} \int_R f(x)K(u,x)dx, & b \neq 0, \\ \sqrt{d}\exp\left(\frac{i}{2}cdu^2\right)f(du), & b = 0, \end{cases} \tag{3}$$

where $K(u,x) = \sqrt{\frac{1}{i2\pi b}}\exp\left\{\frac{i}{2b}[du^2 - 2ux + ax^2]\right\}$, $\mathcal{L}_A$ is the unitary LCT operator, and L^A_f is a LCT of f corresponding to parameter A.

We know consequently that the FT, the FrFT as well as the FRT are particular cases of the LCT. In fact, if we let a parameter $A = [\cos\theta, \sin\theta; -\sin\theta, \cos\theta]$, then the Eq. (3) can be represented as the following

$$\mathcal{L}_A[f](u) = \sqrt{\frac{1}{i2\pi\sin\theta}}\int_R\left[f(x)\exp\left\{\frac{i}{2}[\cot\theta u^2 - 2\csc\theta ux + \cot\theta x^2]\right\}\right]dx.$$

It implies that the LCT contains the FrFT. And if $A = [0,1;-1,0]$, the Eq. (3) simplifies to

$$\mathcal{L}_A[f](u) = \sqrt{\frac{-i}{2\pi}}\int_R[f(x)\exp\{-iux\}]dx,$$

which means that the LCT becomes to the product of the constant $\sqrt{-i}$ and the FT. Moreover, taking $A = [1,b;0,1]$, we can obtain the FRT as follows

$$\mathcal{L}_A[f](u) = \sqrt{\frac{1}{i2\pi b}}\int_R\left[f(x)\exp\left\{\frac{i}{2b}[u^2 - 2ux + x^2]\right\}\right]dx.$$

In the following, we will pay our attention to the LCT for $b \neq 0$.

Let $A = [a_1,b_1;c_1,d_1]$ and $B = [a_2,b_2;c_2,d_2]$, $b_1 \neq 0$, $b_2 \neq 0$. The authors in [38] gave the definition of two-dimensional LCT (short for 2-D LCT) with parameters A and B:

$$L^{A,B}_f(u,v) = \mathcal{L}_{A,B}[f](u,v) = \int_{R^2} f(x,y)K^{A,B}(u,v,x,y)dxdy, \tag{4}$$

where

$$K^{A,B}(u,v,x,y) = \frac{1}{2\pi}\sqrt{\frac{-1}{b_1b_2}}\exp\left\{\frac{i}{2b_1}[d_1u^2 - 2ux + a_1x^2]\right\} \cdot \exp\left\{\frac{i}{2b_2}[d_2v^2 - 2vy + a_2y^2]\right\}.$$

This implies that the 2-D LCT with matrix parameters in $SL(2,R)$ has eight parameters and its degree of freedom is 6. We also know that the two-dimensional FrFT can be reduced from the 2-D LCT when $A = B = [\cos\theta, \sin\theta; -\sin\theta, \cos\theta]$, and the two-dimensional FT can be obtained from the two-dimensional FrFT if $\theta = \frac{\pi}{2}$.

The inverse transform of the 2-D LCT (2-D ILCT) with parameters A^{-1} and B^{-1} is defined as:

$$\mathcal{L}_{A^{-1},B^{-1}}\{L^{A,B}_f\}(x,y) = \int_{R^2} L^{A,B}_f(u,v)K^{A^{-1},B^{-1}}(x,y,u,v)dudv, \tag{5}$$