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Novel efficient two-pass algorithm for closed polygonal approximation based on LISE and curvature constraint criteria

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Abstract

Given a closed curve with n points, based on the local integral square error and the curvature constraint criteria, this paper presents a novel two-pass $O(Fn + mn^2)$-time algorithm for solving the closed polygonal approximation problem where $m(\ll n)$ denotes the minimal number of covering feasible segments for one point and empirically the value of m is rather small, and $F\ (\ll n^2)$ denotes the number of feasible approximate segments. Based on some real closed curves, experimental results demonstrate that under the same number of segments used, our proposed two-pass algorithm has better quality and execution-time performance when compared to the previous algorithm by Chung et al. Experimental results also demonstrate that under the same number of segments used, our proposed two-pass algorithm has better quality, but has some execution-time degradation when compared to the currently published algorithms by Wu and Sarfraz et al.

Keywords: Algorithm; Closed curve; Closed polygonal approximation algorithm; Curvature; Local integral square error; Shortest path algorithm

1. Introduction

Polygonal approximation (PA) is an important method for shape representation [4]. The primary goal of PA is to determine an approximate polygonal curve as the contour representation of one object. Sometimes the approximate polygonal curve can be thought as a special compression method for representing the contour of that object. Given a polygonal curve C with n vertices, $C = \langle P_1, P_2, P_3, \ldots, P_n \rangle$, the PA problem is to find another similar polygon C' with n' vertices, where $C' = \langle P_{1'}, P_{2'}, P_{3'}, \ldots, P_{n'} \rangle$ and $|C'| \leqslant |C|$, such that the obtained approximate polygonal curve satisfies some error criteria to retain an acceptable quality. The determined n' vertices in C' must be a subset of C. Besides the compression benefit, the closed approximate polygonal curve C' can make the manipulation and analysis, e.g., rotation, translation, scaling, clipping, and partitioning, easy and it leads to a computational benefit. In addition, the PA approach has also been applied to the binary image progressive transmission [8] successfully.

In the past three decades, many efficient PA algorithms for open curves have been developed. These developed PA algorithms can be classified into two types, the sub-optimal algorithms [18,14,13,6,7] and the optimal algorithms [10,3,17,12,11,1,9]. These heuristic sub-optimal PA algorithms are quite fast, but the obtained approximate polygonal curves usually are local optimal solutions. To extend the open PA problem to the closed PA (CPA) problem, naturally we exhaustively examine all vertices in C as the possible starting points, and finally determine the global solution within these n possible solutions. For solving the CPA problem, Sato [17] presented an $O(n^3)$-time algorithm in which the selected starting point is the farthest point

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from the center of gravity. The error criterion of arc lengths between the original curve and the approximating curve is used. Based on the L_∞ metric, Zhu and Seneviratne [21] presented a heuristic $O(n^3)$-time algorithm for solving the CPA problem. Horng and Li [5] presented a heuristic $O(sn^2)$-time algorithm where s denotes the number of specified polygonal segments. Since the above three efficient algorithms for solving the CPA problem are heuristic, the optimization of their results can not be guaranteed. In [12], the global integral square error (ISE) criterion is used. In [14,13,1], the local ISE (LISE) criterion is used. In [1], the presented CPA algorithm takes $O(n^3)$ time and the number of polygonal segments determined is minimal. Basically, the LISE metric can keep more peak information when compared to the global ISE. Besides using the LISE error criterion in the CPA problem, the curvature constraint is also a common error criterion. Under the same k-cosine curvature metric, Wu and Wang [19] presented an efficient coarse-to-fine algorithm for determining dominant points which were connected as the solution of the CPA problem.

Based on the concept of break points, Wu [20] presented an adaptive, improved CPA algorithm. In Wu's algorithm, the set of break points are first filtered out using the curvature criterion. Further, the break points with maximum curvature are selected as the dominant points which are considered in his CPA algorithm. In [16], Sarfraz et al. presented a recursive algorithm for solving the CPA problem. Their algorithm extracts initial break points as a preprocessing step. Among these initial break points, they first consider points with angle of 135° as the dominant points in their CPA algorithm. If no point with 135° angle is found, then the points with angle of 90° are considered. If the points with 90° angle are still not available, then the first break point is considered as the dominant point. Experimental results demonstrated that the CPA algorithm by Sarfraz et al. is quite competitive with the one by Wu. The motivation of this paper is to design a novel, efficient CPA algorithm under the three error criteria, namely the LISE, the curvature constraint, and the longest vertical distance consideration.

This paper presents a novel two-pass $O(Fn + mn^2)$-time algorithm for solving the CPA problem where $m(\ll n)$ denotes the minimal number of covering segments for one point and empirically the value of m is rather small and $F(\ll n^2)$ denotes the number of feasible approximate segments. According to the concept of covering segments for each point, the first pass of our proposed algorithm can be performed in $O(Fn + mn^2)$ time under the given LISE criterion; the second pass of our proposed algorithm can be performed in $O(n)$ time under the given curvature constraint and the longest vertical distance consideration. Because of $Fn + mn^2 \ll n^3$ and considering these criteria, our proposed CPA algorithm has better time performance and quality when compared to the previous CPA algorithm by Chung et al. [1] which takes $O(n^3)$ time complexity and considers only the LISE criterion. Based on two real closed curves for representing French and Italy, experimental results demonstrate that under the same number of segments used, our proposed two-pass algorithm has better quality and execution-time performance when compared to the currently published algorithm by Chung et al. [1]. Based on two real closed curves for representing a semicircle and a chromosome, experimental results demonstrate that under the same number of segments used, our proposed two-pass algorithm has better quality, but has some execution-time degradation when compared to currently published algorithms by Wu [20] and Sarfraz et al. [16].

The rest of this paper is organized as follows. Section 2 introduces the concerned two error criteria, the LISE bound and the curvature constraint. Section 3 presents new methods for determining all feasible segments and covering segment sets which will be used in our proposed two-pass CPA algorithm. Section 4 presents our proposed whole two-pass CPA algorithm. The definition of longest vertical distance consideration will be explained in Section 4. Experimental results are demonstrated in Section 5. Conclusions are addressed in Section 6.

2. Error criteria

In this section, both the LISE and the curvature criteria are introduced. For exposition, the next paragraph introduces the definition of discrete curvature measure, which will be used in the second pass of our proposed CPA algorithm, and how to measure it for each point on the original curve. The curvature constraint accompanied with the longest vertical distance consideration will be introduced in Section 4.2. Next, the definition of LISE is given and it will be used in the first pass of our proposed CPA algorithm.

Curvature can be explained as how much the curve bends at each point on the curve. It has been defined that the original polygonal curve is presented by the set $\{P_i = (x_i, y_i) | i = 1, 2, \ldots, n\}$ and P_i denotes the ith point with coordinate (x_i, y_i) on the original polygonal curve. Following the k-cosine value to estimate the curvature of each point, as shown in Fig. 1, the estimated curvature at point P_i related to two points P_{i-k} and P_{i+k}, is set to be the k-cosine value cos_{ik}.

Definition 1. [15] The k-cosine value at point P_i related to two k-index apart neighboring points P_{i-k} and P_{i+k} is defined by

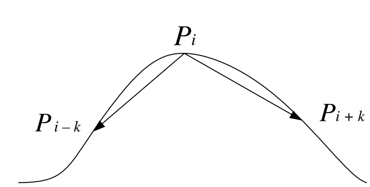

Fig. 1. The depiction for the definition of k-cosine value.